Endüstriyel Uygulamalarda Kullanılan Yüksek İvmeli Çalışan Paralel Robotların Konumlama Hassasiyetinin Artırılmasına Yönelik Yöntemler Çalıştayı

-Direngenlik Modeli-

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Makine Mühendisliği Bölümü
Modelleme ve Prototipleme Laboratuvarı

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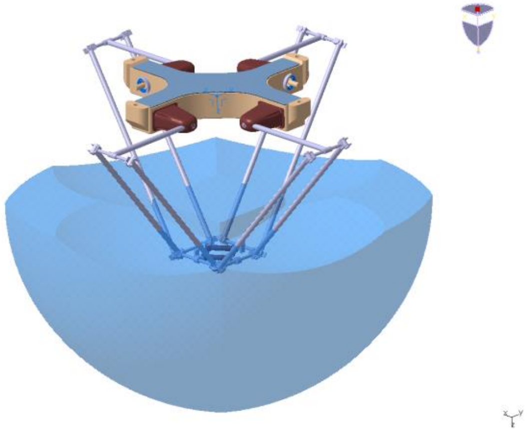
Bu çalışma 116M272 nolu proje kapsamında TÜBİTAK tarafından finanse edilmektedir.

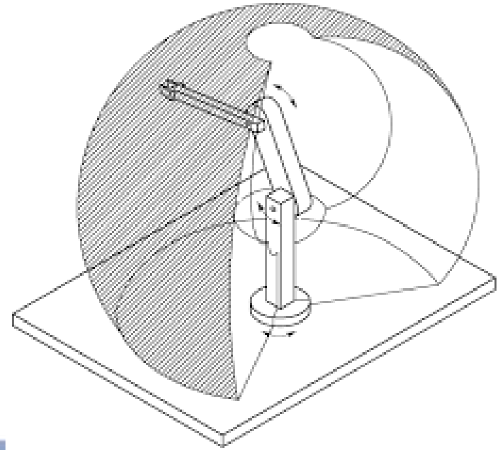
Robot destekli/temelli üretim.

- Esnek üretime izin verirler.
- Boyutlarına oranla çok büyük çalışma alanları vardır.
- Hafiftirler. Yüksek hızda hareket edebilirler.
- Akıllıdırlar.
- Paralel, seri, ya da hibrit (paralel+seri) manipülatörler üzerine kurulabilirler.

Tek bir robot kol farklı ürünleri ve farklı işlemleri gerçekleştirebilir. Başka işler için programlanabilir.

Operatör olmadan karar verebilir, hataları düzeltebilirler.





Seri ve paralel robotlar.

- Seri robotlarda uzuvlar birbirlerinin uç kısımlarından bağlanır ve her uzuvda hareketi sağlayan motor bulunur.
- Paralel robotlarda motorlar şase üzerindedir ve her seri zincir hareketli platformda birbirlerine bağlanır.

	Seri Robotlar	Paralel robotlar	CNC
Çalışma uzayı	Yüksek	Orta	Kısıtlı
Üretim Esnekliği	Yüksek	Yüksek	Kısıtlı
Hassasiyet	Direngenlik, kütle, çalışma hızı vb. durumlara bağlı		Yüksek
Dinamik Performans (Atalet)	Orta	Yüksek	Düşük
Direngenlik	Orta	Yüksek	Yüksek
Montaj Hataları	Her bağlantıda artar.	Her bağlantıda azalır.	-

Uç eyleyici konumlandırılması.

- İleri kinematik + enkoder bilgisi kullanılır. (Teorik olarak hesaplanır.)

Uç eyleyici konumunun doğru hesaplanamaması.

Geometrik Hatalar (Baskın):

- Uzuv boyutu (Üretim hatası.)
- Montaj hataları (eksen kaçıklığı vb.)
- Mafsal hataları (Mafsal boşluğu.)

Kinematik modelin güncellenmesi, tasarımın iyileştirilmesi, üretim hassasiyetinin artırılması.

Geometrik Olmayan Hatalar:

- Esneme
- Çevresel faktörler (ısıl genleşme vb.)
- Aşınma, sürtünme
- Kontrolcü hataları

Normal çalışma şartlarında konumlama hassasiyetini en fazla etkileyen faktör.

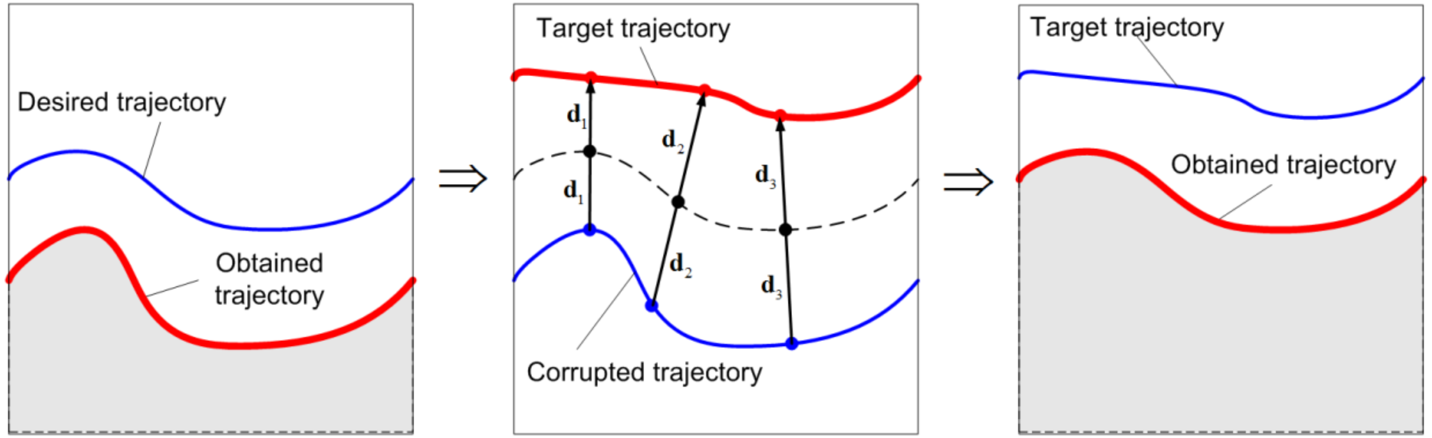
Uç eyleyici konumu dışarıdan kamera sistemiyle bulunabilir ancak bu pahalı ve pratik olmayan bir çözümdür.

Daha ucuz ve pratik çözüm doğru bir matematiksel model elde etmek.

Esneme modelinin kullanılması

Çevrim dışı
(takım yolu düzeltmesi)

$$F = K\Delta x$$



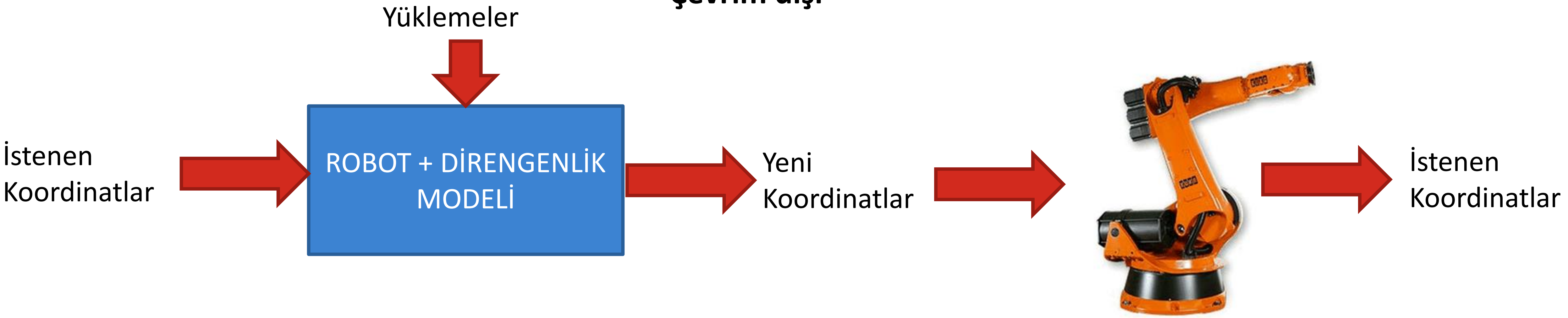
(a) machining experiment (b) compensation algorithm (c) final machining

Çevrimiçi
Anlık Düzeltme

Yada kuvvetin anlık hesaplanması /ölçülmesi → Esneme miktarının hesaplanıp düzeltilmesi.

Esneme modelinin kullanılması

Çevrim dışı



Yüklemeler

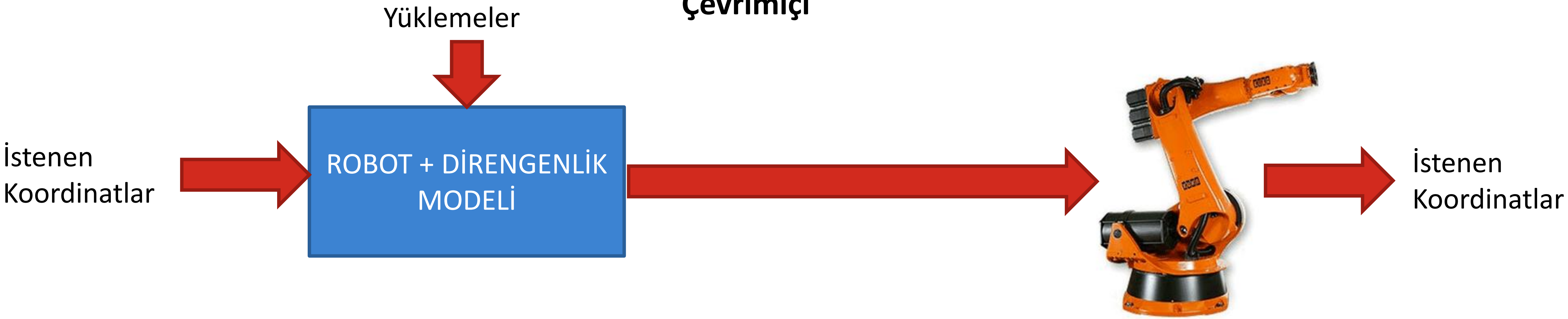
İstenen Koordinatlar

ROBOT + DİRENGENLİK MODELİ

Yeni Koordinatlar

İstenen Koordinatlar

Çevrimiçi



Yüklemeler

İstenen Koordinatlar

ROBOT + DİRENGENLİK MODELİ

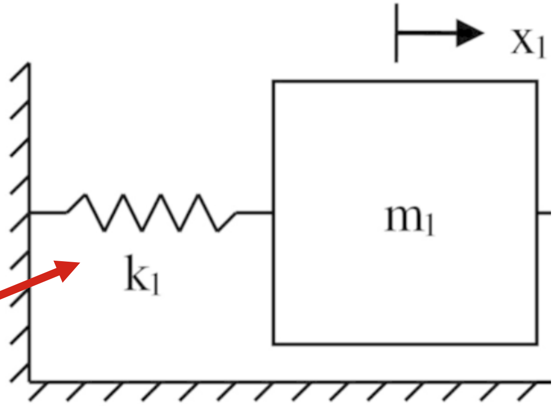
İstenen Koordinatlar

Direngenlik Modeli

Esnek uzva gelen kuvvet. ← $F = K\Delta x$ → **Esneme miktarı**

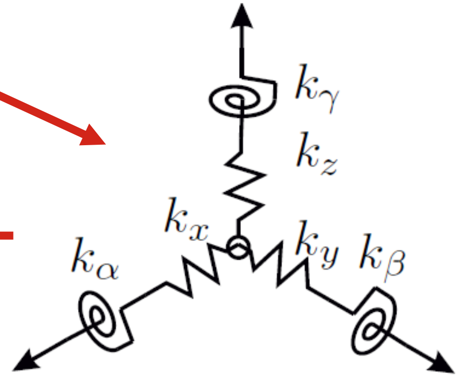
↓

Direngenlik modeli.

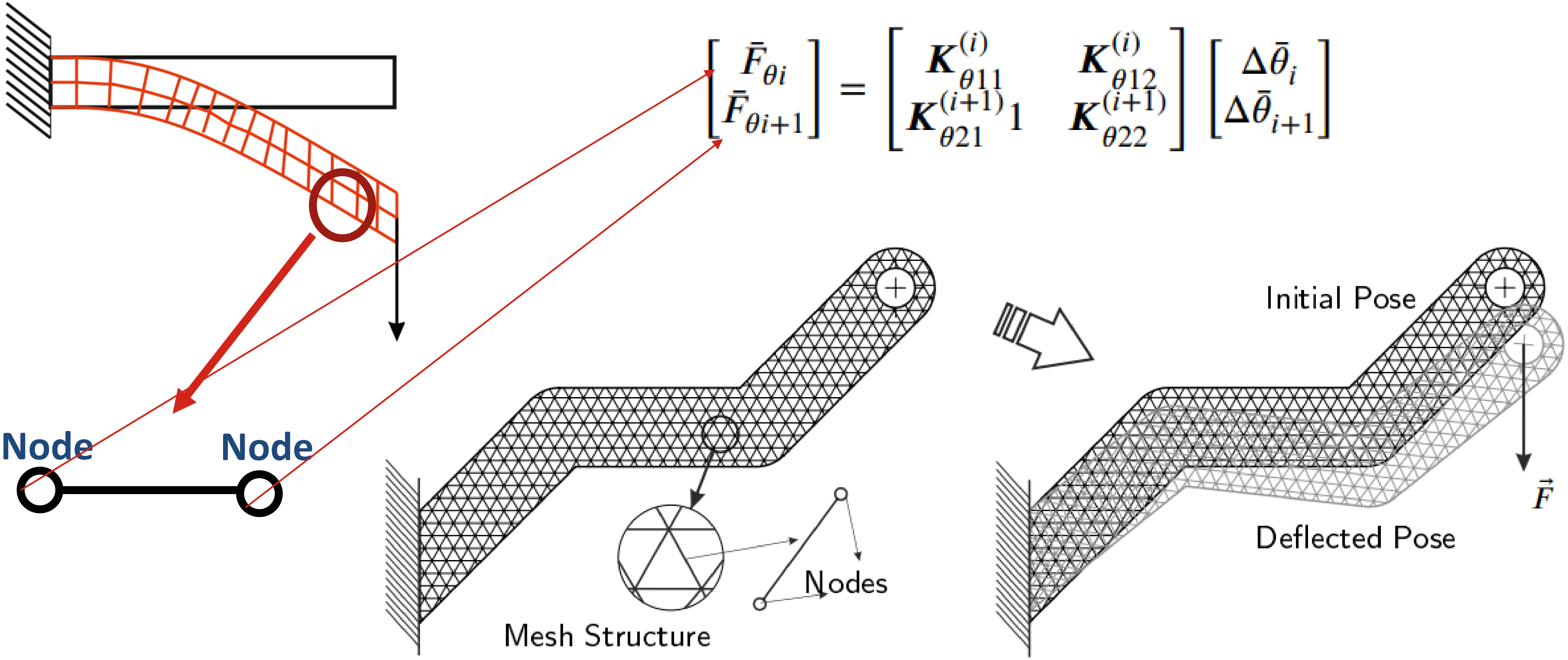


- **İdeal bir yay ve bir serbestlik derecesi için K sabittir. (Lineer fonksiyon.)**
- **Ancak esneme 6 boyutlu uzayda her yönde gerçekleşebilir. (3 dönel+3 öteleme)**

$$\mathbf{K} = \frac{E}{L^3} \cdot \begin{bmatrix} A \cdot L^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 12 \cdot I_z & 0 & 0 & 0 & -6 \cdot I_z \cdot L \\ 0 & 0 & 12 \cdot I_y & 0 & 6 \cdot I_y \cdot L & 0 \\ 0 & 0 & 0 & G \cdot J \cdot L^2 / E & 0 & 0 \\ 0 & 0 & 6 \cdot I_y \cdot L & 0 & 4 \cdot I_y \cdot L^2 & 0 \\ 0 & -6 \cdot I_z \cdot L & 0 & 0 & 0 & 4 \cdot I_z \cdot L^2 \end{bmatrix}$$



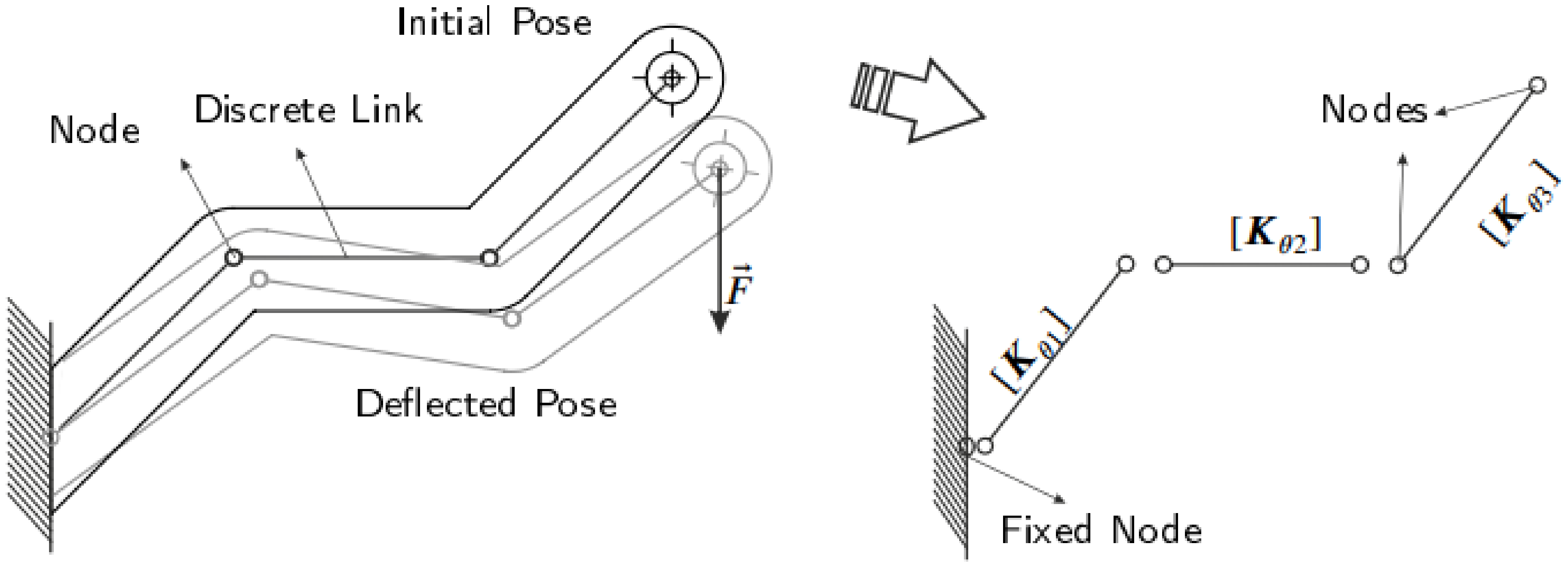
Sonlu Eleman Analizi (Finite Element Method-FEM)



$$\begin{bmatrix} \bar{F}_{\theta i} \\ \bar{F}_{\theta i+1} \end{bmatrix} = \begin{bmatrix} \boldsymbol{K}^{(i)}_{\theta 11} & \boldsymbol{K}^{(i)}_{\theta 12} \\ \boldsymbol{K}^{(i+1)}_{\theta 21}1 & \boldsymbol{K}^{(i+1)}_{\theta 22} \end{bmatrix} \begin{bmatrix} \Delta\bar{\theta}_i \\ \Delta\bar{\theta}_{i+1} \end{bmatrix}$$

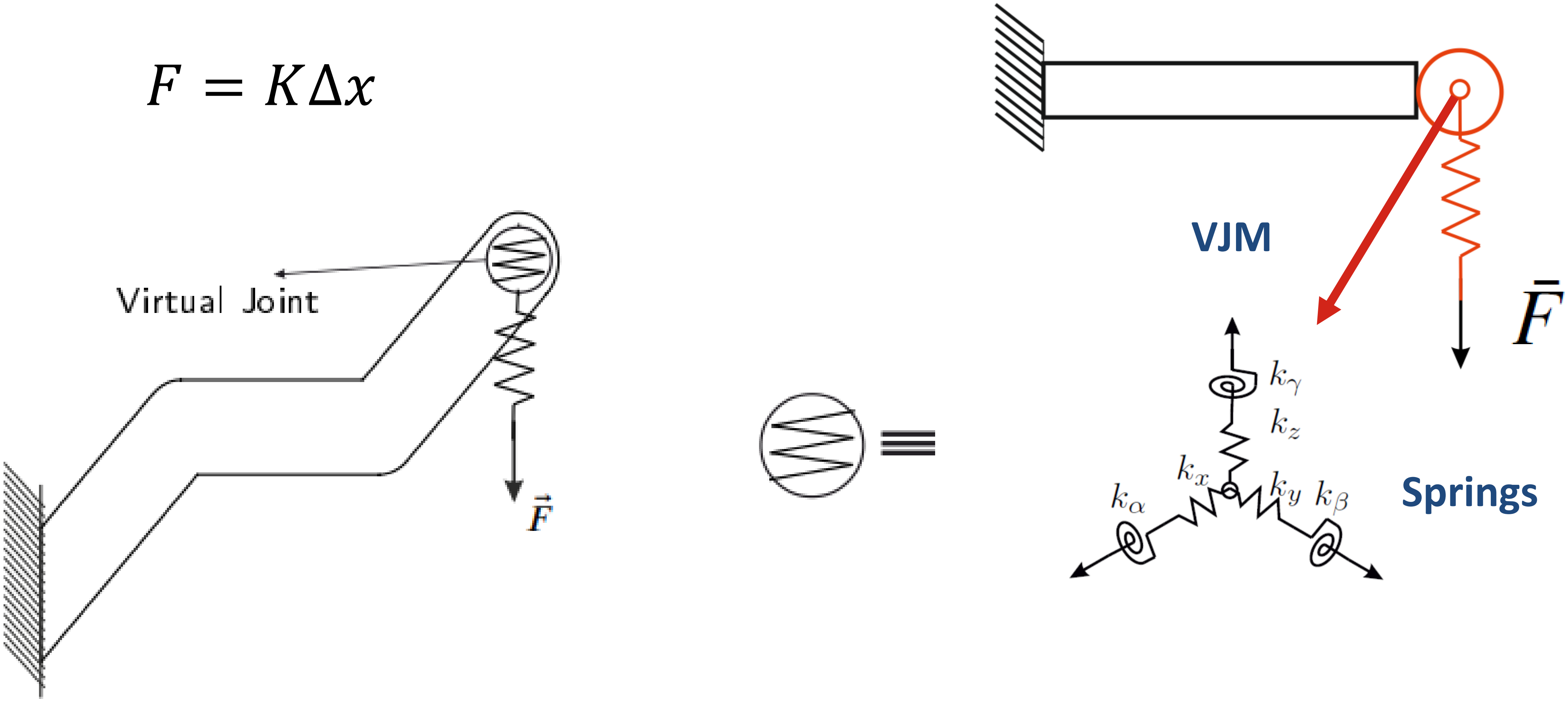
Yapısal Matris Analizi (Structural Matrix Method-SMM)

$$\begin{bmatrix} \bar{F}_{\theta i} \\ \bar{F}_{\theta i+1} \end{bmatrix} = \begin{bmatrix} \boldsymbol{K}^{(i)}_{\theta 11} & \boldsymbol{K}^{(i)}_{\theta 12} \\ \boldsymbol{K}^{(i+1)}_{\theta 21} 1 & \boldsymbol{K}^{(i+1)}_{\theta 22} \end{bmatrix} \begin{bmatrix} \Delta\bar{\theta}_i \\ \Delta\bar{\theta}_{i+1} \end{bmatrix}$$



Sanal Mafsal Yöntemi (Virtual Joint Method-VJM)

$$F = K\Delta x$$



Gerilme Enerjisi Yöntemi (Strain Energy Method-SEM)

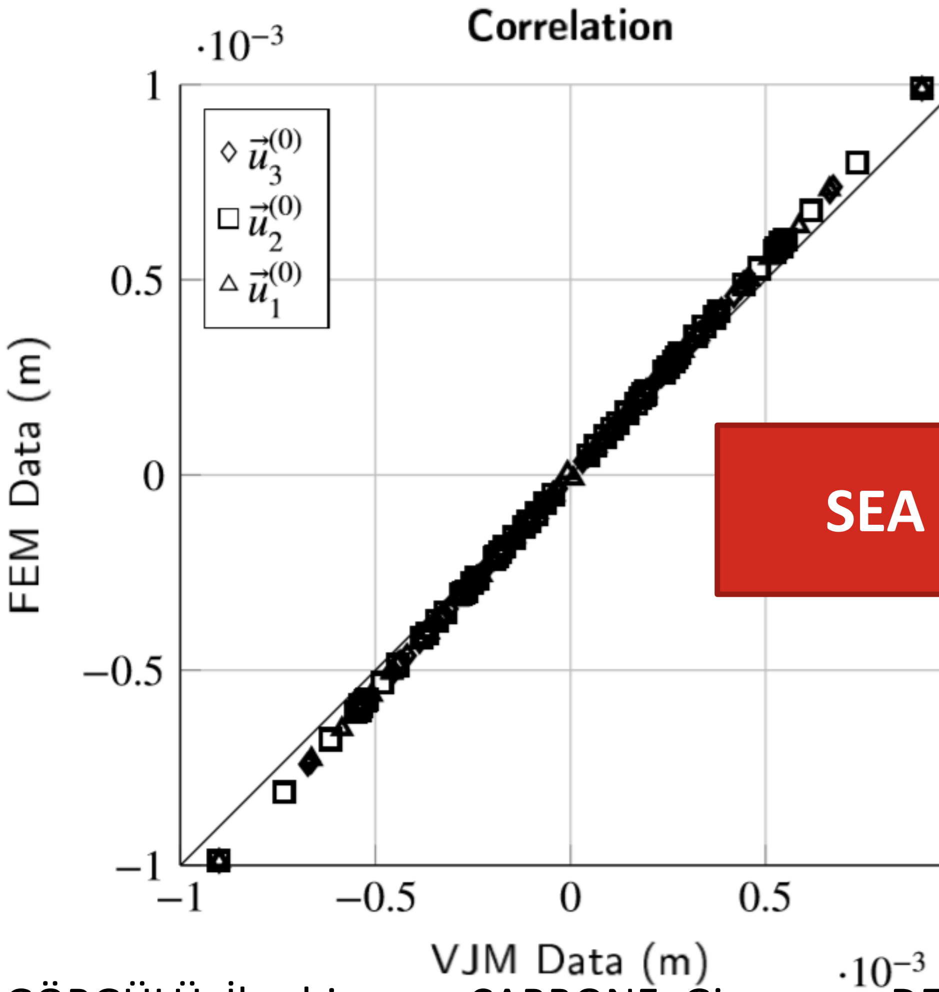
$$F = K\Delta x$$
$$C = K^{-1}$$
$$CF = \Delta x$$

$$U_C = \frac{1}{2}\Delta\bar{F}_{ext}^{T} C_C \Delta\bar{F}_{ext}, \quad C_C = \frac{\partial^2 U}{\partial \bar{F}_{ext}^2}$$

$$U_\theta = \frac{1}{2}\Delta\bar{F}_{\theta}^{T} C_\theta \Delta\bar{F}_{\theta}, \quad C_\theta = \frac{\partial^2 U_\theta}{\partial \bar{F}_{\theta}^2}$$

	SEA	YMA	SMY
Doğruluk	Yüksek	Orta	Düşük
Hesaplama Süresi	Uzun	Orta	Kısa

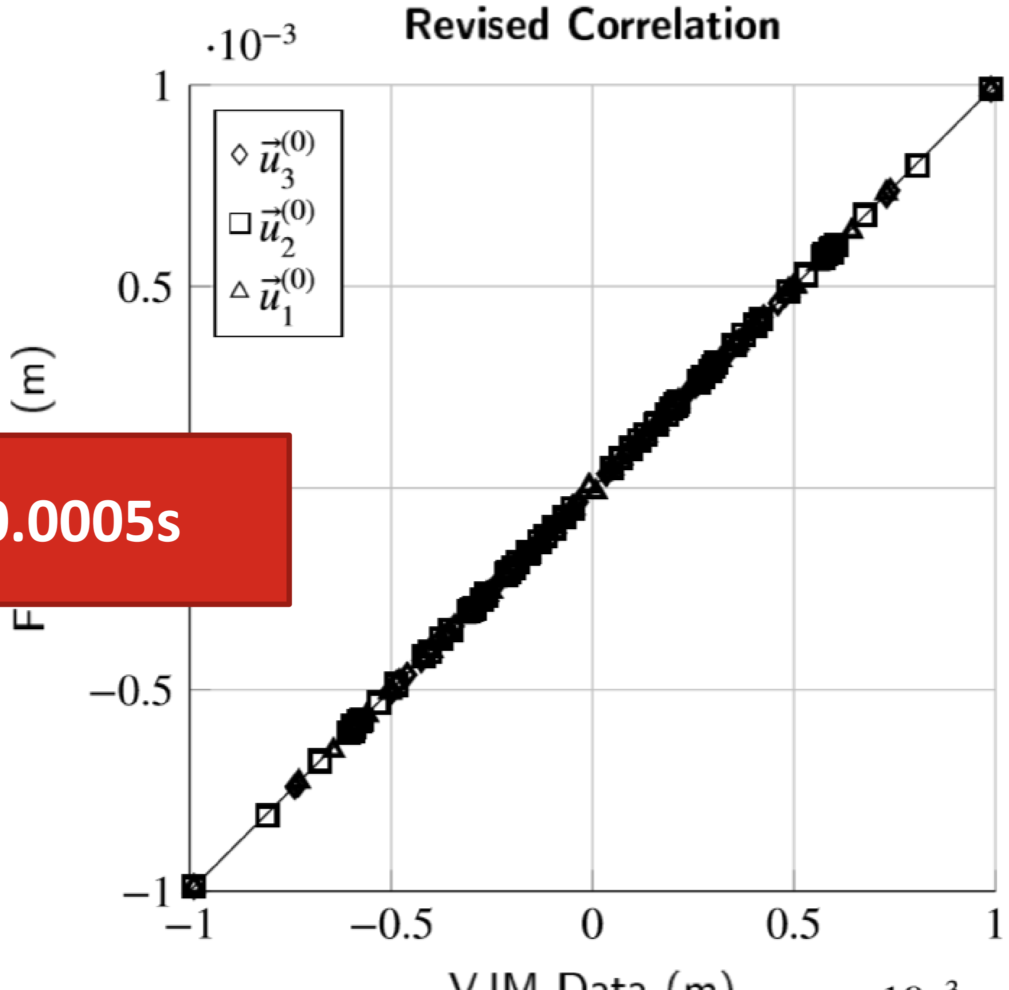
SMY ve SEA karşılaştırma



~%10 Hata

SEA 24s – SMY 0.0005s

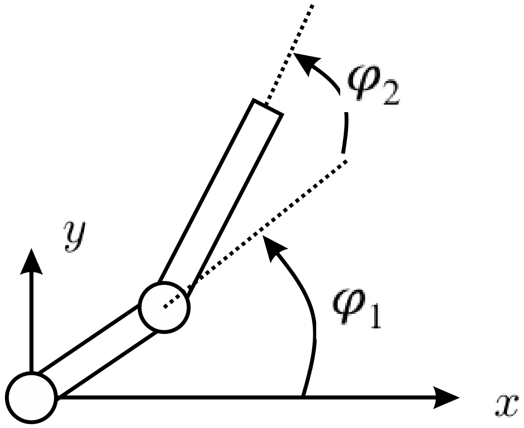
~%1 Hata

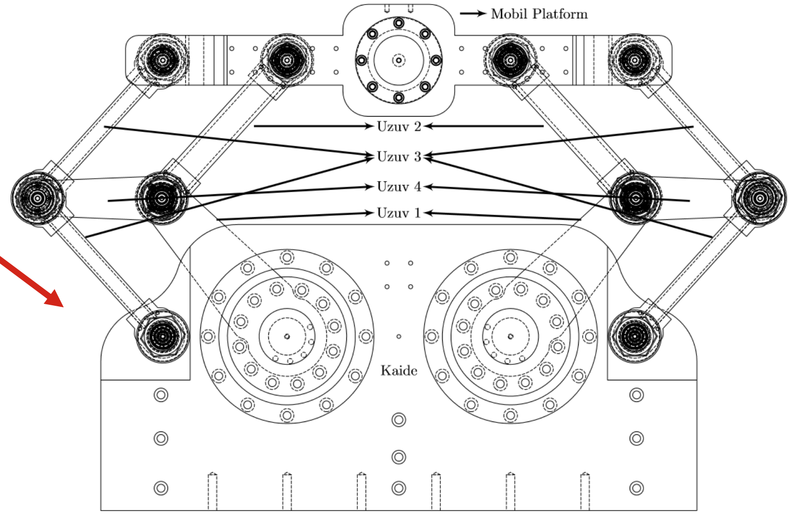


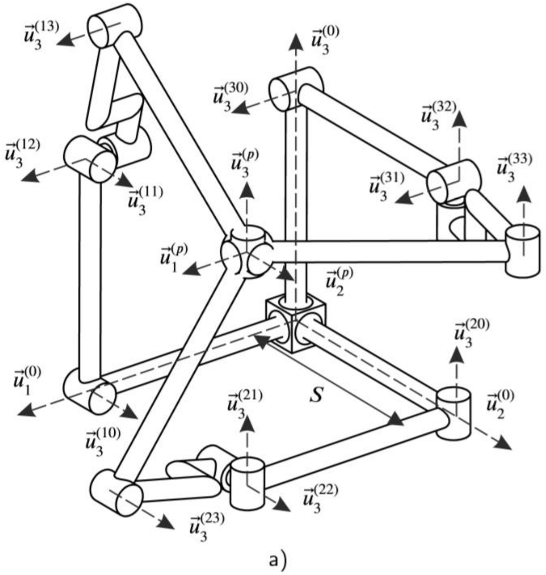
GÖRGÜLÜ, İbrahimcan; CARBONE, Giuseppe; DEDE, Mİ Can. Time efficient stiffness model computation for a parallel haptic mechanism via the virtual joint method. *Mechanism and Machine Theory*, 2020, 143: 103614.

Direngenlik Modeli Oluşturma

- 2 serbestlik dereceli seri manipülatör. (Sadece aktif mafsallar.)
- 3 serbestlik dereceli paralel manipülatör. (Kısıtlılanmış kinematik. Aktif+ pasif mafsallar.)
- Paralel Lazer kesim makinesi manipülatörü. (Fazla Kısıtlılanmış kinematik. Aktif+pasif mafsallar)

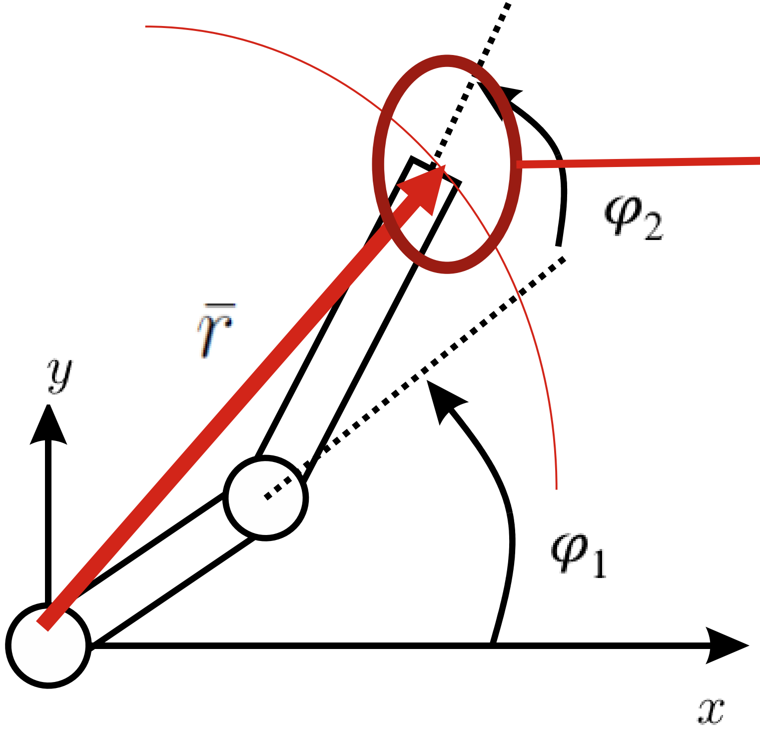






Sanal Mafsal Yöntemi (Virtual Joint Method-VJM) ile modelleme

Kinematik



$$\bar{q} = \begin{bmatrix} \varphi_1 & \varphi_2 \end{bmatrix}^T_{2\times1}$$

$$x = l_1 cos(q_1) + l_2 cos(q_1 + q_2)$$

$$y = l_1 sin(q_1) + l_2 sin(q_1 + q_2)$$

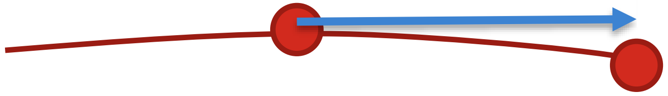
$$\bar{r} = \bar{f}(\bar{q})$$

$$\frac{\mathrm{d}\bar{r}}{\mathrm{d}t} = \frac{\partial \bar{f}(\bar{q})}{\partial \bar{q}} \frac{\mathrm{d}\bar{q}}{\mathrm{d}t}$$

$$\dot{\bar{r}} = \hat{J}\dot{\bar{q}}$$

Hız katsayı matrisi= Jakobi Matrisi

Uç eyleyici konum değişimi: $\dot{\bar{r}}$

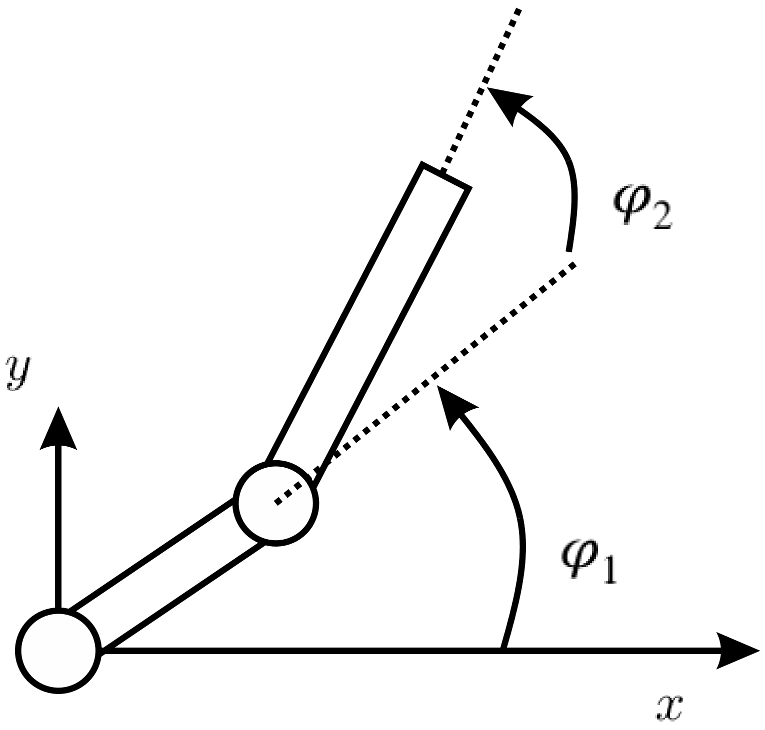


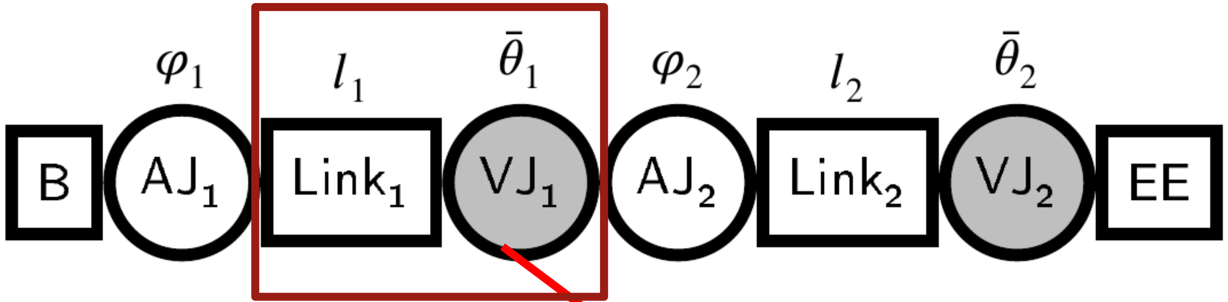
$$\Delta\bar{r} = f(\Delta\bar{q})$$

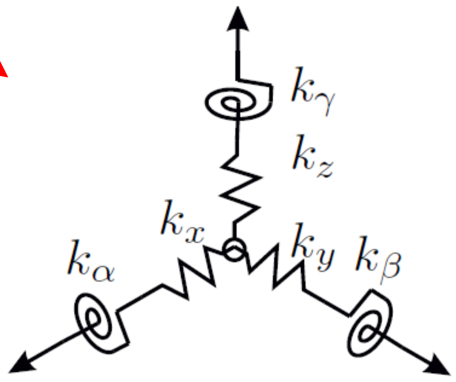
Küçük yer değişimi için:

$$\Delta\bar{r} \approx \boldsymbol{J}\Delta\bar{q}$$

SMY 2 serbestlik dereceli manipülatör



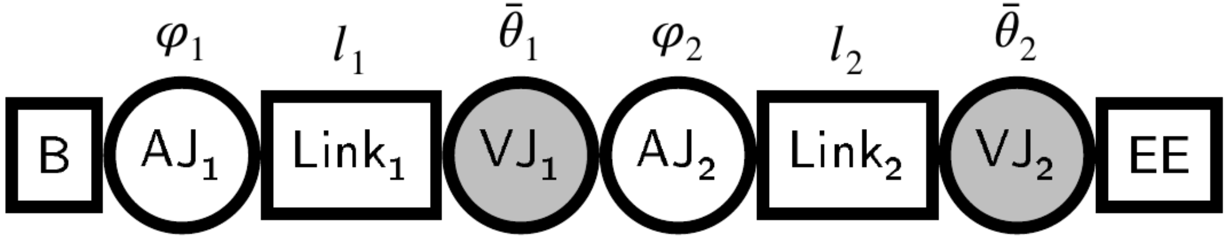




$$\bar{\theta}_i = \left[\underbrace{\theta_i^1\ \ \theta_i^2\ \ \theta_i^3}_{\text{Öteleme}}\ \ \underbrace{\theta_i^4\ \ \theta_i^5\ \ \theta_i^6}_{\text{Dönme}}\right]^T$$

$$\boldsymbol{H}_v(\bar{\theta}) = \boldsymbol{T}_1(\theta^1)\boldsymbol{T}_2(\theta^2)\boldsymbol{T}_3(\theta^3)\boldsymbol{R}_1(\theta^4)\boldsymbol{R}_2(\theta^5)\boldsymbol{R}_3(\theta^6)$$

SMY 2 serbestlik dereceli manipülatör



$$\bar{q} = \begin{bmatrix} \varphi_1 & \varphi_2 \end{bmatrix}^T_{2\times 1},$$

$$\bar{\theta} = \begin{bmatrix} \bar{\theta}_1^T & \bar{\theta}_2^T \end{bmatrix}^T_{12\times 1}$$

$$\bar{Q} = \begin{bmatrix} \bar{\theta}^T & \bar{q}^T \end{bmatrix}^T_{14\times 1}$$

$$\boldsymbol{H}_v(\bar{\theta}) = \boldsymbol{T}_1(\theta^1)\boldsymbol{T}_2(\theta^2)\boldsymbol{T}_3(\theta^3)\boldsymbol{R}_1(\theta^4)\boldsymbol{R}_2(\theta^5)\boldsymbol{R}_3(\theta^6)$$

Esneme bilgisini içeren ileri kinematik modeli:

$$\boldsymbol{H}_K = \boldsymbol{R}_3(\varphi_1)\boldsymbol{T}_1(l_1)\boldsymbol{H}_v(\bar{\theta}_1)\boldsymbol{R}_3(\varphi_2)\boldsymbol{T}_1(l_2)\boldsymbol{H}_v(\bar{\theta}_2)$$

$$\frac{\partial \boldsymbol{H}_K}{\partial Q_k} = \begin{bmatrix} \dfrac{\partial \boldsymbol{R}_K}{\partial Q_k} & \dfrac{\partial \bar{r}_K}{\partial Q_k} \\ \bar{0}^T & 1 \end{bmatrix}$$

$$\boldsymbol{H}_K = \begin{bmatrix} \boldsymbol{R}_K & \bar{r}_K \\ \bar{0}^T & 1 \end{bmatrix}$$

$$\bar{J}_{Kk} = \begin{bmatrix} \bar{V}_k^T & \bar{\Omega}_k^T \end{bmatrix}^T$$

SMY 2 serbestlik dereceli manipülatör

Aktif mafsalların uç eyleyiciye etkisi:

$$\boldsymbol{J}_a = \begin{bmatrix} \bar{J}_{K13} & \bar{J}_{K14} \end{bmatrix}_{6\times 2}$$

Sanal mafsalların uç eyleyiciye etkisi:

$$\boldsymbol{J}_\theta = \begin{bmatrix} \bar{J}_{K1} & \bar{J}_{K2} & \cdots & \bar{J}_{Ki12} \end{bmatrix}_{6\times 12}$$

Toplam etki:

$$\boldsymbol{J}_K = \begin{bmatrix} \boldsymbol{J}_\theta & \boldsymbol{J}_a \end{bmatrix}_{6\times 14}$$

$$\Delta\bar{X} = \boldsymbol{J}_K \Delta\bar{Q}$$

$$\Delta\bar{X} = \boldsymbol{J}_\theta \Delta\bar{\theta} + \bar{\boldsymbol{J}}_a \Delta q$$

Mafsallardaki kuvvet ve torklar (momentler).

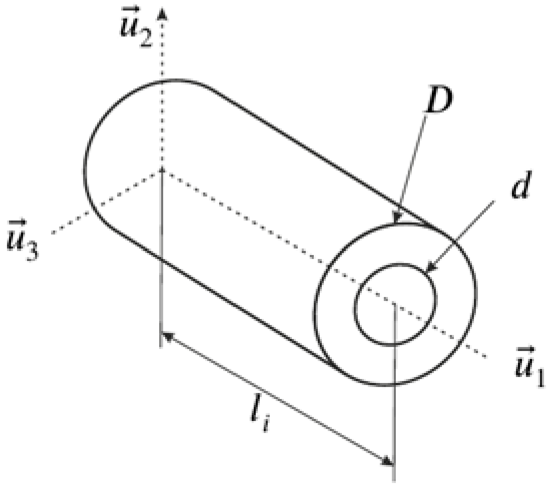
$$\bar{F}_K = \begin{bmatrix} \bar{F}_\theta^T & \bar{F}_a^T \end{bmatrix}^T \quad [\bar{F}_\theta]_{12\times 1}, [\bar{F}_a]_{2\times 1}$$

Uç eyleyiciye gelen kuvvetlerin mafsallara dağıtılması:

$$\boxed{\bar{F}_K = \boldsymbol{J}_K^T \bar{F}_{ext}}$$

$$\bar{F}_K = \begin{bmatrix} \bar{F}_\theta^T & \bar{F}_a^T \end{bmatrix}^T = \begin{bmatrix} \boldsymbol{J}_\theta & \boldsymbol{0} \end{bmatrix}^T \bar{F}_{ext} + \begin{bmatrix} \boldsymbol{0} & \boldsymbol{J}_a \end{bmatrix}^T \bar{F}_{ext}$$

SMY 2 serbestlik dereceli manipülatör



Esneme ve kuvvet ilişkisi (çalışma uzayında):

$$\bar{F}_{ext} = K_C \Delta \bar{X}$$

Uç eyleyici esneme miktarı. -Aranan-

Çalışma uzayında tanımlı direngenlik modeli. -Bilinmiyor-

Manipülatöre gelen kuvvet. -Biliniyor-

Esneme ve kuvvet ilişkisi (mafsal uzayında):

$$\bar{F}_K = K_\theta \Delta \bar{\theta}$$

Mafsal esneme miktarı. -Aranan-

Mafsal uzayında tanımlı direngenlik modeli. -Biliniyor-

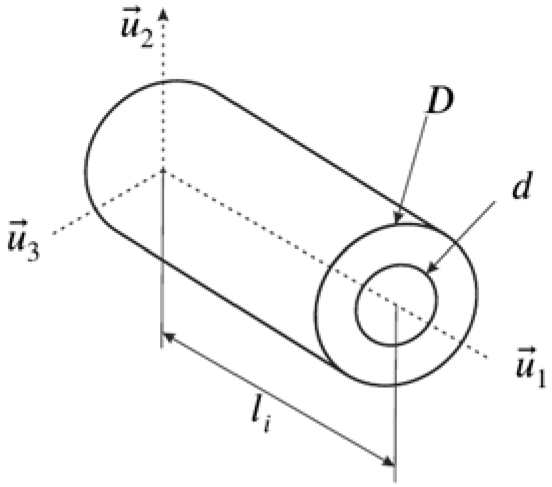
Mafsallara gelen kuvvet. -Biliniyor-

$$\mathbf{K} = \frac{E}{L^3} \cdot \begin{bmatrix} A \cdot L^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 12 \cdot I_z & 0 & 0 & 0 & -6 \cdot I_z \cdot L \\ 0 & 0 & 12 \cdot I_y & 0 & 6 \cdot I_y \cdot L & 0 \\ 0 & 0 & 0 & G \cdot J \cdot L^2 / E & 0 & 0 \\ 0 & 0 & 6 \cdot I_y \cdot L & 0 & 4 \cdot I_y \cdot L^2 & 0 \\ 0 & -6 \cdot I_z \cdot L & 0 & 0 & 0 & 4 \cdot I_z \cdot L^2 \end{bmatrix}$$

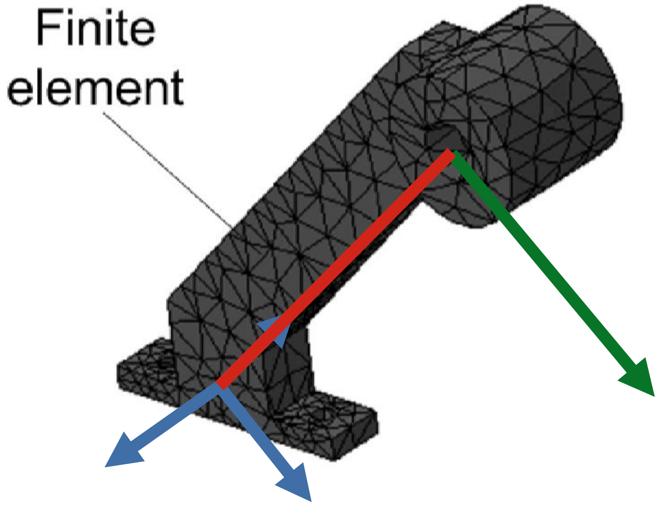
SMY 2 serbestlik dereceli manipülatör

Mafsal uzayında direngenlik matrisini belirleme: $\boldsymbol{K}_{\theta}$

1. Basit uzuvlar içir analitik olarak:



2. Sonlu eleman yöntemi ile:



3. Deneysel:

- CMM
- Lazer mesafe sensörü
- Komparatör

$$\frac{E}{L^3}\cdot\begin{bmatrix} A\cdot L^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 12\cdot I_z & 0 & 0 & 0 & -6\cdot I_z\cdot L \\ 0 & 0 & 12\cdot I_y & 0 & 6\cdot I_y\cdot L & 0 \\ 0 & 0 & 0 & G\cdot J\cdot L^2/E & 0 & 0 \\ 0 & 0 & 6\cdot I_y\cdot L & 0 & 4\cdot I_y\cdot L^2 & 0 \\ 0 & -6\cdot I_z\cdot L & 0 & 0 & 0 & 4\cdot I_z\cdot L^2 \end{bmatrix}$$

SMY 2 serbestlik dereceli manipülatör

Esneme ve kuvvet ilişkisi (mafsal uzayında):

$$\bar{F}_K = diag(\boldsymbol{K}_\theta, K_a)\Delta\bar{Q}$$

$$\boldsymbol{K}_\theta = diag(\boldsymbol{K}_{\theta 1}, \boldsymbol{K}_{\theta 2})_{12\times 12}$$

Bilinenler:

$$\bar{F}_K = \boldsymbol{J}_K^T \bar{F}_{ext}$$

$$\Delta\bar{X} = \boldsymbol{J}_K \Delta\bar{Q}$$

$$\boldsymbol{J}_K^{-1}\Delta\bar{X} = \Delta\bar{Q}$$

$$\boldsymbol{J}_K^T \bar{F}_{ext} = diag(\boldsymbol{K}_\theta, K_a)\Delta\bar{Q}$$

$$\bar{F}_{ext} = \boldsymbol{J}_K^{-T} diag(\boldsymbol{K}_{\theta 1}, \boldsymbol{K}_{\theta 2}, \boldsymbol{K}_a)\boldsymbol{J}_K^{-1}\Delta\bar{X}$$

$$\bar{F}_{ext} = \underbrace{(\boldsymbol{J}_\theta^{-T}\boldsymbol{K}_\theta\boldsymbol{J}_\theta^{-1} + \bar{J}_a^{-T}K_a\bar{J}_a^{-1})}_{\boldsymbol{K}_C}\Delta\bar{X}$$

SMY 2 serbestlik dereceli manipülatör

Eyleyiciler (Aktif mafsallar) rijit kabul edilirse:

$$\boldsymbol{K}_a = \infty$$

$$\Delta q \approx 0$$

Sanal mafsalın ilk değeri

$$\Delta\bar{\theta} = \bar{\theta}_{son} - \bar{\theta}_{ilk}$$

$$\bar{\theta}_{ilk} = 0$$

$$\Delta\bar{\theta} = \bar{\theta}_{son}$$

$$\Rightarrow \Delta\bar{X} = \boldsymbol{J}_\theta\bar{\theta}$$

Yeni denklem:

$$\bar{F}_{ext} = (\boldsymbol{J}_\theta^{-T}\boldsymbol{K}_\theta\boldsymbol{J}_\theta^{-1})\Delta\bar{X}$$

$$\bar{F}_{ext} = \boldsymbol{J}_\theta^{-T}\boldsymbol{K}_\theta\bar{\theta}$$

$$\boldsymbol{J}_\theta = \boldsymbol{J}_\theta(\bar{\theta}, \bar{q})$$

Enkoder verisi

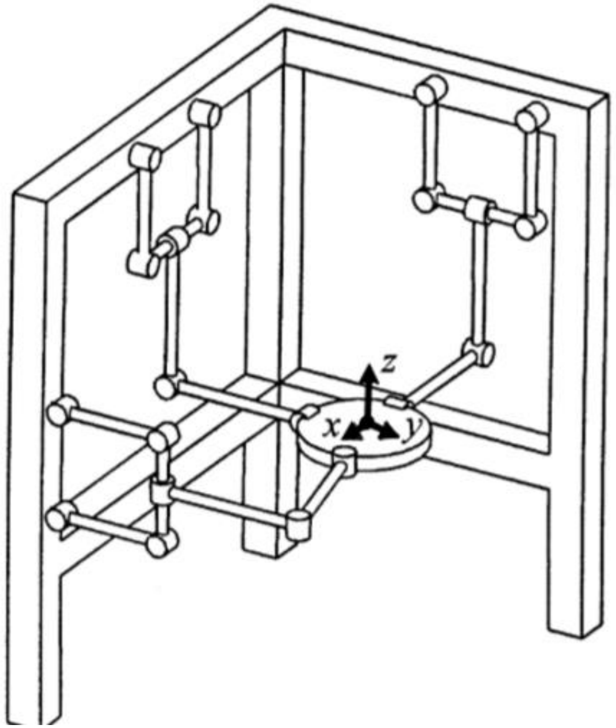
=0

$$\bar{\theta}_{j+1} = \boldsymbol{K}_\theta^{-1}\boldsymbol{J}_\theta^T(\bar{\theta}_j, \bar{q}_j)\bar{F}_j^T + \bar{\theta}_j$$

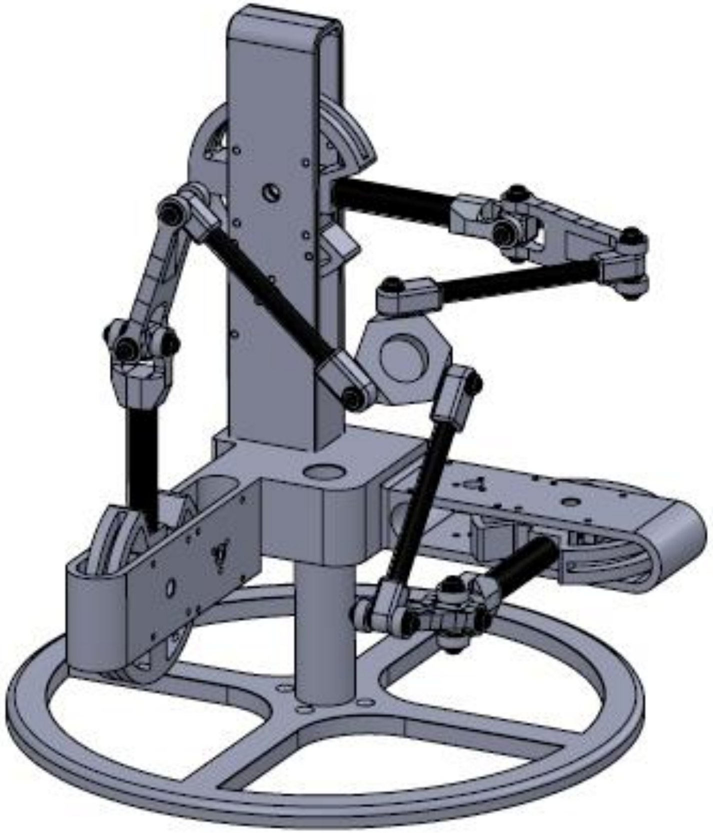
$$\boldsymbol{C}_C = \boldsymbol{K}_C^{-1}$$

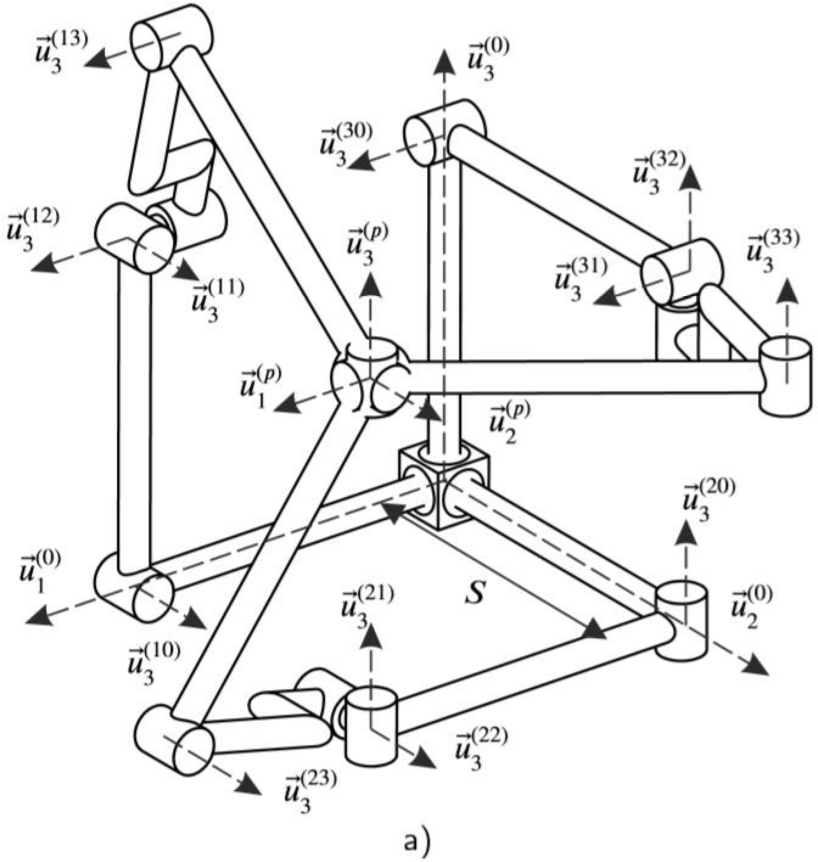
$$\Delta\bar{X} = \boldsymbol{C}_C\bar{F}_{ext}.$$

SMY 3 serbestlik dereceli paralel manipülatör

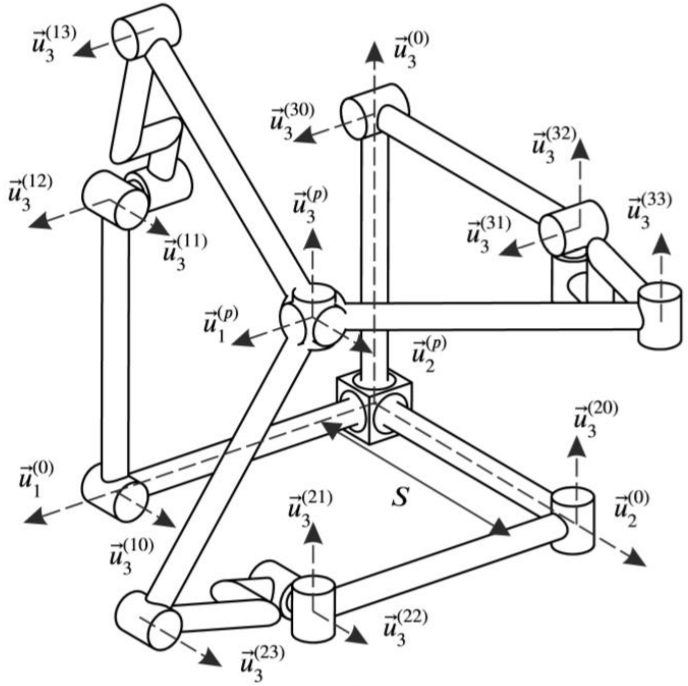


R-CUBE

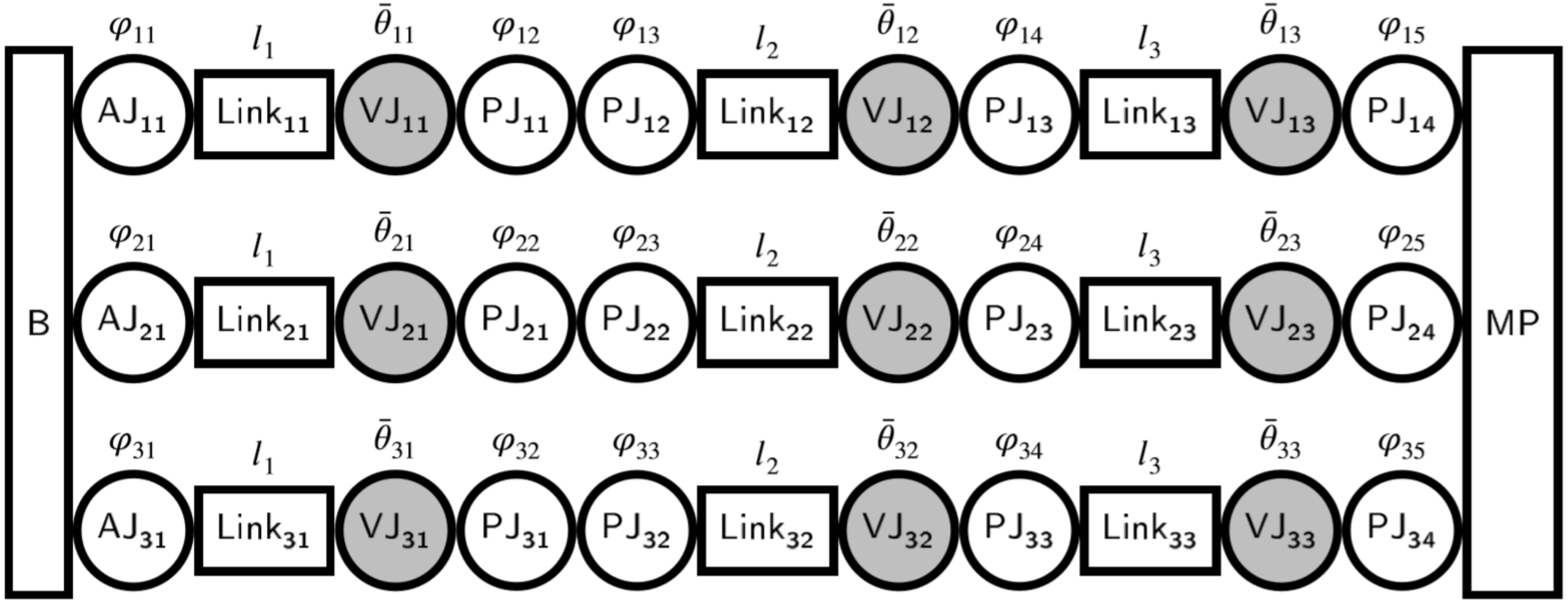




SMY 3 serbestlik dereceli paralel manipülatör



a)



SMY 3 serbestlik dereceli paralel manipülatör

$$\boldsymbol{H}^{(i0,K_{i1})} = \boldsymbol{R}_3(\varphi_{i1})\boldsymbol{T}_1(l_1)\boldsymbol{H}_v(\bar{\theta}_{i1})$$

$$\boldsymbol{H}^{(K_{i1},K_{i2})} = \boldsymbol{R}_3(\varphi_{i2})\boldsymbol{R}_1(-\pi/2)\boldsymbol{R}_3(\varphi_{i3})\boldsymbol{T}_1(l_2)\boldsymbol{H}_v(\bar{\theta}_{i2})$$

$$\boldsymbol{H}^{(K_{i2},K_{i3})} = \boldsymbol{R}_3(\varphi_{i4})\boldsymbol{T}_1(l_3)\boldsymbol{H}_v(\bar{\theta}_{i3})$$

$$\boldsymbol{H}^{(K_{i3},i5)} = \boldsymbol{R}_3(\varphi_{i5})$$

$$\boldsymbol{H}_{Ki} = \boldsymbol{H}^{(0,i0)}\boldsymbol{H}^{(i0,K_{i1})}\boldsymbol{H}^{(K_{i1},K_{i2})}\boldsymbol{H}^{(K_{i2},K_{i3})}\boldsymbol{H}^{(K_{i3},i5)}\boldsymbol{H}^{(i5,p)}$$

$$\boldsymbol{H}_{Ki} = \begin{bmatrix} \boldsymbol{R}_{Ki} & \bar{r}_{Ki} \\ \bar{0}^T & 1 \end{bmatrix}$$

$$\boldsymbol{H}_{K1} = \boldsymbol{H}_{K2} = \boldsymbol{H}_{K3}$$

$$\bar{q}_{pi} = \begin{bmatrix} \varphi_{i2} & \varphi_{i3} & \varphi_{i4} & \varphi_{i5} \end{bmatrix}^T_{4\times1}, \quad \bar{q}_i = \varphi_{i1}$$

$$\bar{\theta}_i = \begin{bmatrix} \bar{\theta}^T_{i1} & \bar{\theta}^T_{i2} & \bar{\theta}^T_{i3} \end{bmatrix}^T_{18\times1}$$

$$\bar{Q}_i = \begin{bmatrix} \bar{\theta}^T_i & \bar{q}^T_{pi} & q_{i1} \end{bmatrix}^T_{23\times1}$$

Sanal mafsal etkisi: $\boldsymbol{J}_{\theta i} = \begin{bmatrix} \bar{\boldsymbol{J}}_{Ki1} & \bar{\boldsymbol{J}}_{Ki2} & \dots & \bar{\boldsymbol{J}}_{Ki18} \end{bmatrix}_{6\times18}$

Pasif mafsal etkisi: $\boldsymbol{J}_{pi} = \begin{bmatrix} \bar{\boldsymbol{J}}_{Ki19} & \bar{\boldsymbol{J}}_{Ki20} & \bar{\boldsymbol{J}}_{Ki21} & \bar{\boldsymbol{J}}_{Ki22} \end{bmatrix}_{6\times4}$

Aktif mafsal etkisi: $\bar{\boldsymbol{J}}_{ai} = \begin{bmatrix} \bar{\boldsymbol{J}}_{Ki23} \end{bmatrix}_{6\times1}$

$$\boldsymbol{J}_{Ki} = \begin{bmatrix} \boldsymbol{J}_{\theta i} & \boldsymbol{J}_{pi} & \bar{\boldsymbol{J}}_{ai} \end{bmatrix}_{6\times23}$$

SMY 3 serbestlik dereceli paralel manipülatör

$$\bar{F}_{ext} = (\boldsymbol{J}_{\theta i}^{-T}\boldsymbol{K}_{\theta i}\boldsymbol{J}_{\theta i}^{-1} + \boldsymbol{J}_{pi}^{-T}\boldsymbol{K}_{pi}\boldsymbol{J}_{pi}^{-1} + \bar{J}_{ai}^{-T}K_{ai}\bar{J}_{ai}^{-1})\Delta\bar{X}_i$$

Aktif mafsallar rijit.

$$\bar{F}_{ext} = (\boldsymbol{J}_{\theta i}^{-T}\boldsymbol{K}_{\theta i}\boldsymbol{J}_{\theta i}^{-1} + \boldsymbol{J}_{pi}^{-T}\boldsymbol{K}_{pi}\boldsymbol{J}_{pi}^{-1})\Delta\bar{X}_i$$

Pasif mafsal direngenliği yok. Kendi eksenlerinde dönme serbestliği var.

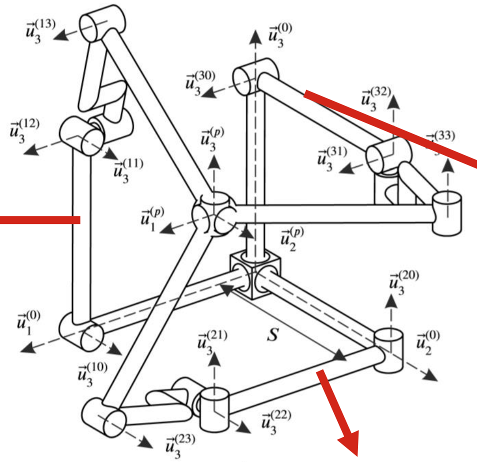
$$\bar{F}_{ext} = (\boldsymbol{J}_{\theta i}^{-T}\boldsymbol{K}_{\theta i}\boldsymbol{J}_{\theta i}^{-1})\Delta\bar{X}_i \qquad \boldsymbol{J}_{pi}^{T}\bar{F}_{ext} = \bar{0}$$

Pasif eksen direngenlik matrisindeki bazı değerlerin sıfırlanmasına yol açar. Bu etki şöyle hesaplanır.

$$\begin{bmatrix} (\boldsymbol{J}_{\theta i}\boldsymbol{K}_{\theta i}^{-1}\boldsymbol{J}_{\theta i}^{T}) & \boldsymbol{J}_{pi} \\ \boldsymbol{J}_{pi}^{T} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \bar{F}_{ext} \\ \Delta\bar{q}_{pi} \end{bmatrix} = \begin{bmatrix} \Delta\bar{X}_i \\ \bar{0} \end{bmatrix} \Longrightarrow \begin{bmatrix} (\boldsymbol{J}_{\theta i}\boldsymbol{K}_{\theta i}^{-1}\boldsymbol{J}_{\theta i}^{T}) & \boldsymbol{J}_{pi} \\ \boldsymbol{J}_{pi}^{T} & \boldsymbol{0} \end{bmatrix}^{-1} = \left[\begin{array}{c|c} [\boldsymbol{K}_{Ci}]_{6\times 6} & \sim \\ \hline \sim & \sim \end{array}\right]$$

SMY 3 serbestlik dereceli paralel manipülatör

$$\hat{K}_{C1} = \begin{bmatrix} K_{C1}11 & 0 & 0 & 0 & K_{C1}15 & K_{C1}16 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ K_{C1}15 & 0 & 0 & 0 & K_{C1}55 & K_{C1}56 \\ K_{C1}16 & 0 & 0 & 0 & K_{C1}56 & K_{C1}66 \end{bmatrix}$$



a)

$$\hat{K}_{C3} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & K_{C3}33 & K_{C3}34 & K_{C3}35 & 0 \\ 0 & 0 & K_{C3}34 & K_{C3}44 & K_{C3}45 & 0 \\ 0 & 0 & K_{C3}35 & K_{C3}45 & K_{C3}55 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

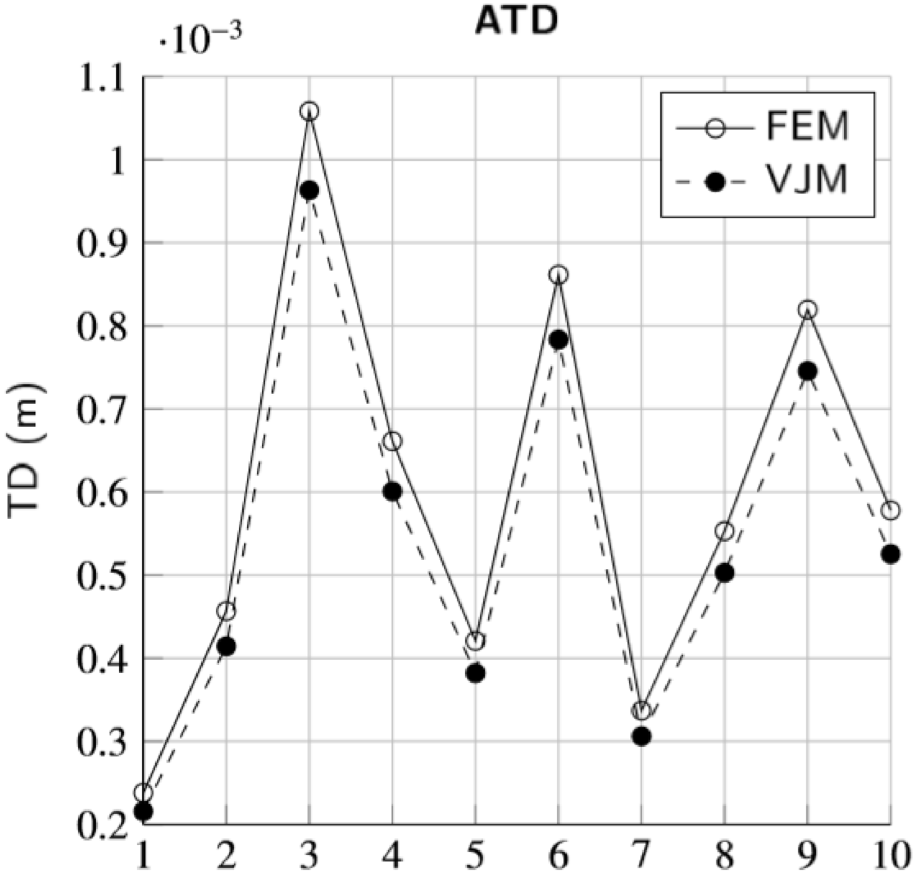
$$\hat{K}_{C2} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \\ 0 & K_{C2}22 & 0 & K_{C2}24 & 0 & K_{C2}26 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & K_{C2}24 & 0 & K_{C2}44 & 0 & K_{C2}46 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & K_{C2}26 & 0 & K_{C2}46 & 0 & K_{C2}66 \end{bmatrix}$$

SMY 3 serbestlik dereceli paralel manipülatör

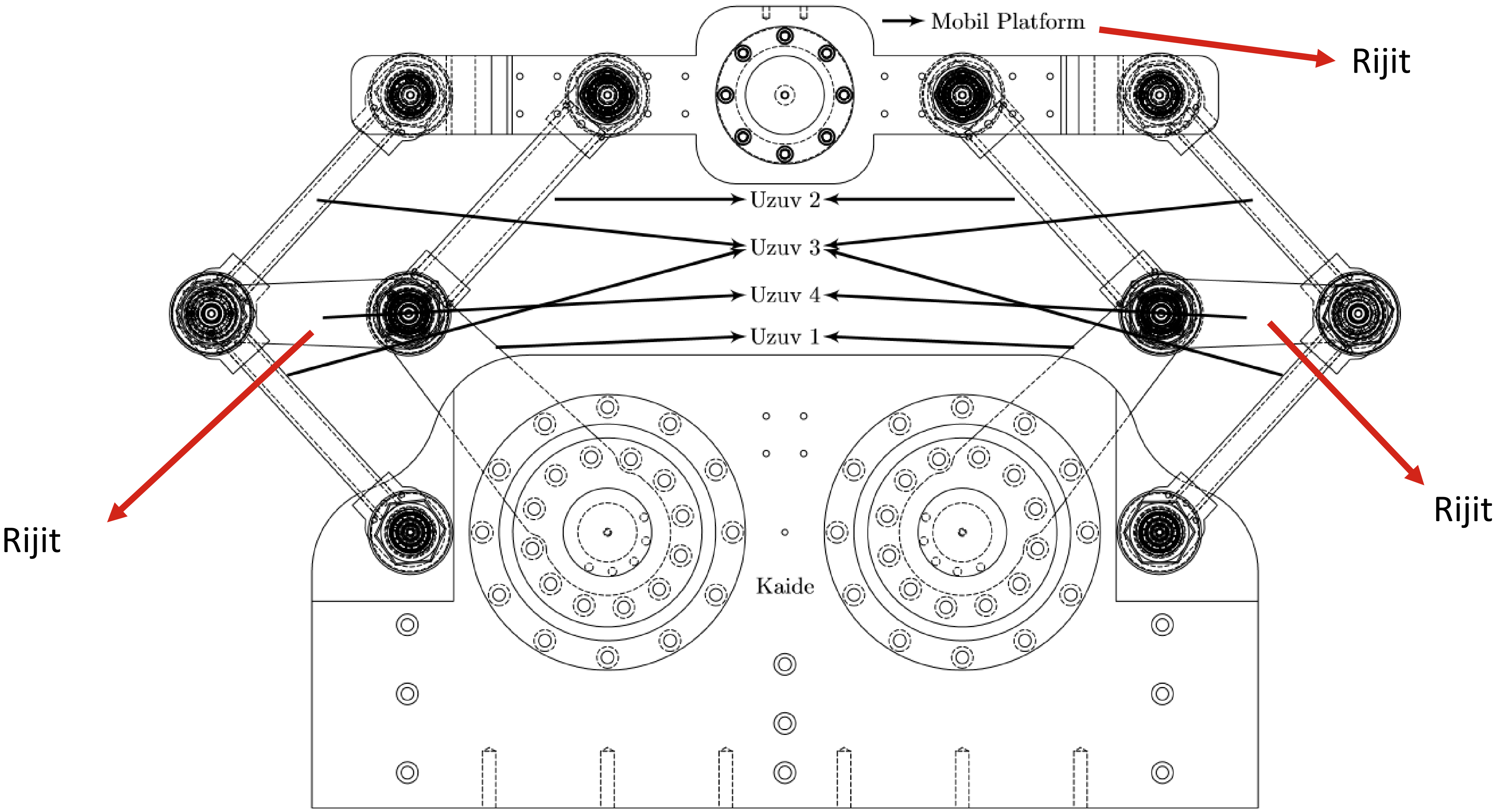
Paralel Yaylar Toplanır: $$\boldsymbol{K}_C = \sum_{i=1}^{3} \boldsymbol{K}_{Ci}$$

$$\hat{K}_C = \begin{bmatrix} K_{C1}11 & 0 & 0 & 0 & K_{C1}15 & K_{C1}16 \\ 0 & K_{C2}22 & 0 & K_{C2}24 & 0 & K_{C2}26 \\ 0 & 0 & K_{C3}33 & K_{C3}34 & K_{C3}35 & 0 \\ 0 & K_{C2}24 & K_{C3}34 & K_{C2}44 + K_{C3}44 & K_{C3}45 & K_{C2}46 \\ K_{C1}15 & 0 & K_{C3}35 & K_{C3}45 & K_{C1}55 + K_{C3}55 & K_{C1}56 \\ K_{C1}16 & K_{C2}26 & 0 & K_{C2}46 & K_{C1}56 & K_{C1}66 + K_{C2}66 \end{bmatrix}$$

GÖRGÜLÜ, İbrahimcan; CARBONE, Giuseppe; DEDE, Mİ Can. Time efficient stiffness model computation for a parallel haptic mechanism via the virtual joint method. *Mechanism and Machine Theory*, 2020, 143: 103614.

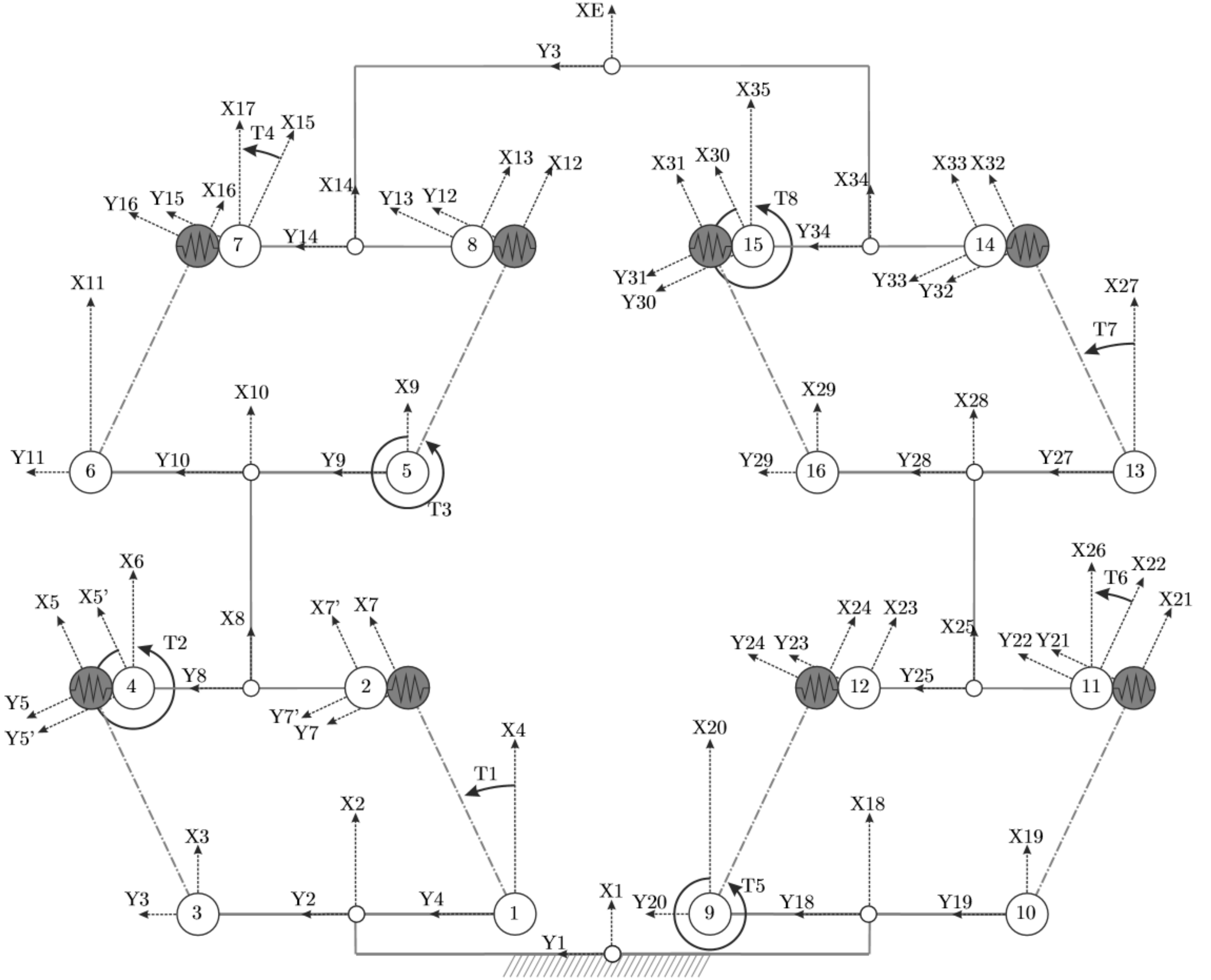


SMY’nin Fazla Kısıtlı Düzlemsel Bir Paralel Mekanizmaya Uygulanması



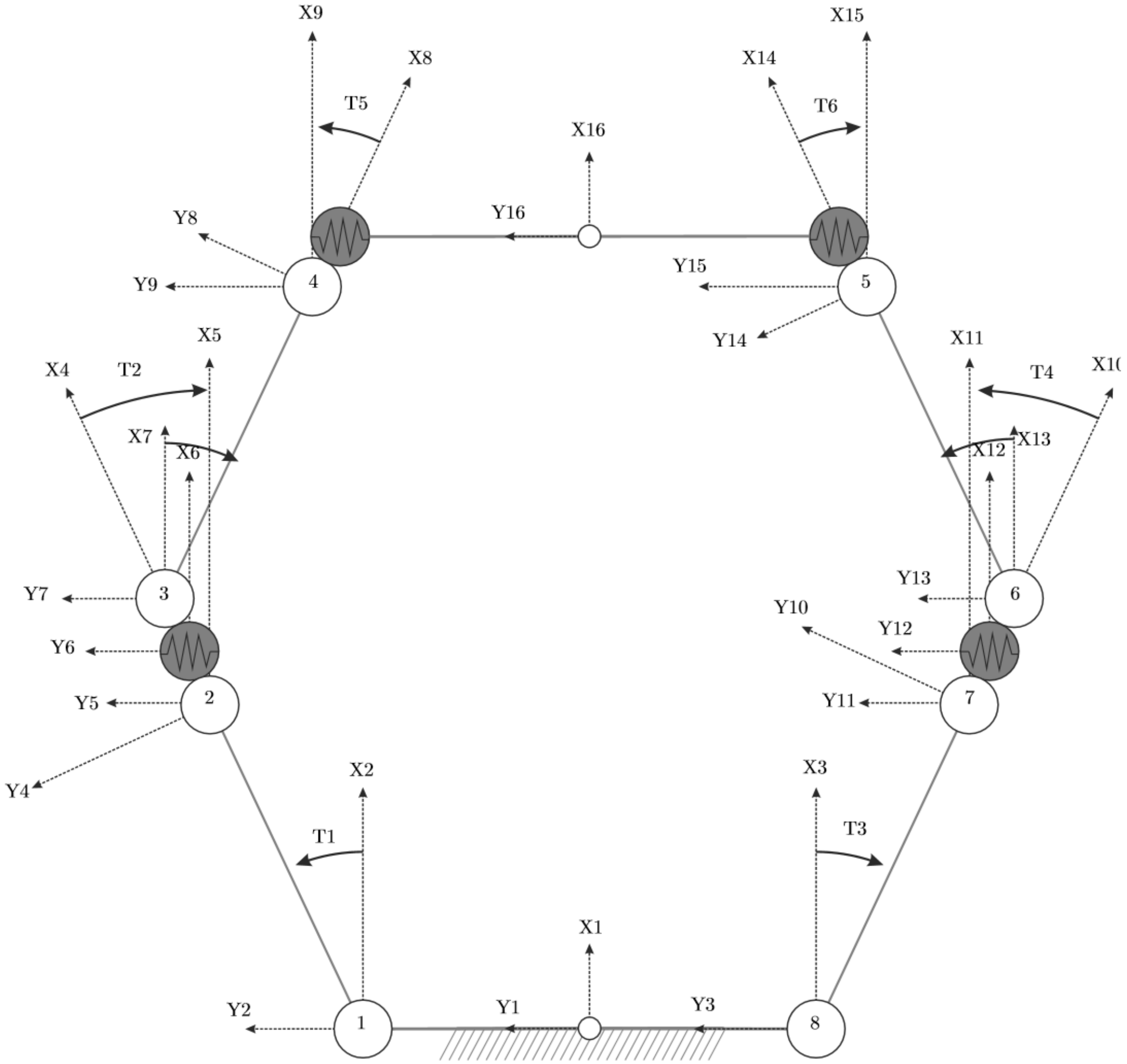
SMY'nin Fazla Kısıtlı Düzlemsel Bir Paralel Mekanizmaya Uygulanması

1. Paralelogramları tek bir uzva dönüştür.
2. Her seri zincir için direngenlik hesapla.
3. Manipülatör direngenliğini hesapla.

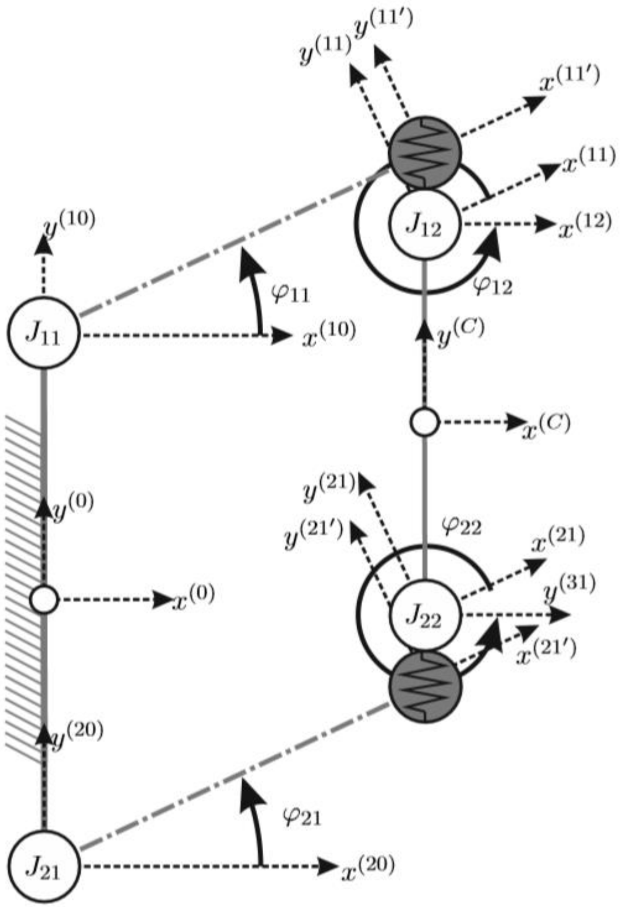


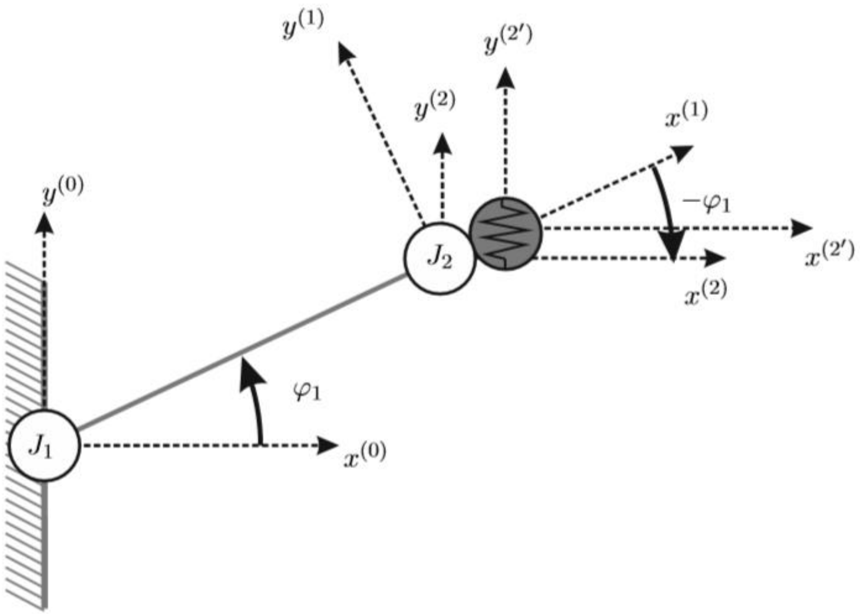
SMY’nin Fazla Kısıtlı Düzlemsel Bir Paralel Mekanizmaya Uygulanması

1. Paralelogramları tek bir uzva dönüştür.
2. Her seri zincir için direngenlik hesapla.
3. Manipülatör direngenliğini hesapla.



SMY’nin Fazla Kısıtlı Düzlemsel Bir Paralel Mekanizmaya Uygulanması

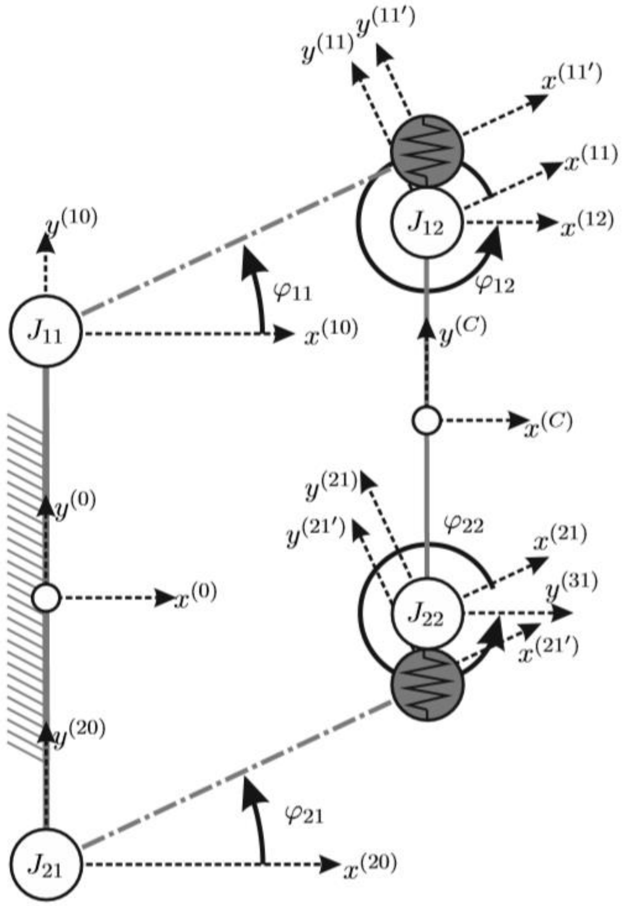




1. Paralelogramları tek bir uzva dönüştür.
2. Her seri zincir için direngenlik hesapla.
3. Manipülatör direngenliğini hesapla.

NOT: Paralelogram direngenlik matrisleri tam dereceli matrisler olmalı. Aksi taktirde paralelogram direngenliği hesaplanırken, tersi alınamayan matrisler oluşur.

SMY’nin Fazla Kısıtlı Düzlemsel Bir Paralel Mekanizmaya Uygulanması (aktif paralelogramlar)



$$\boldsymbol{H}_{UP} = \boldsymbol{T}_2(d/2)\boldsymbol{R}_3(\varphi_{11})\boldsymbol{T}_1(l_1)\boldsymbol{H}_v(\bar{\theta}_{11'})\boldsymbol{R}_3(\varphi_{12})\boldsymbol{T}_2(-d/2)$$

$$\boldsymbol{H}_{DN} = \boldsymbol{T}_2(-d/2)\boldsymbol{R}_3(\varphi_{21})\boldsymbol{T}_1(l_1)\boldsymbol{H}_v(\bar{\theta}_{21'})\boldsymbol{R}_3(\varphi_{22})\boldsymbol{T}_2(d/2)$$

$$\boldsymbol{H}_{UP} = \boldsymbol{H}_{DN}$$

$$\varphi_{11} = \varphi_{21}, \quad \varphi_{12} = \varphi_{22}, \quad \varphi_{11} + \varphi_{12} = \varphi_{21} + \varphi_{22} = 2\pi k \ \text{ for } \ k = 1, 2, ..., \infty$$

Değişkenler:

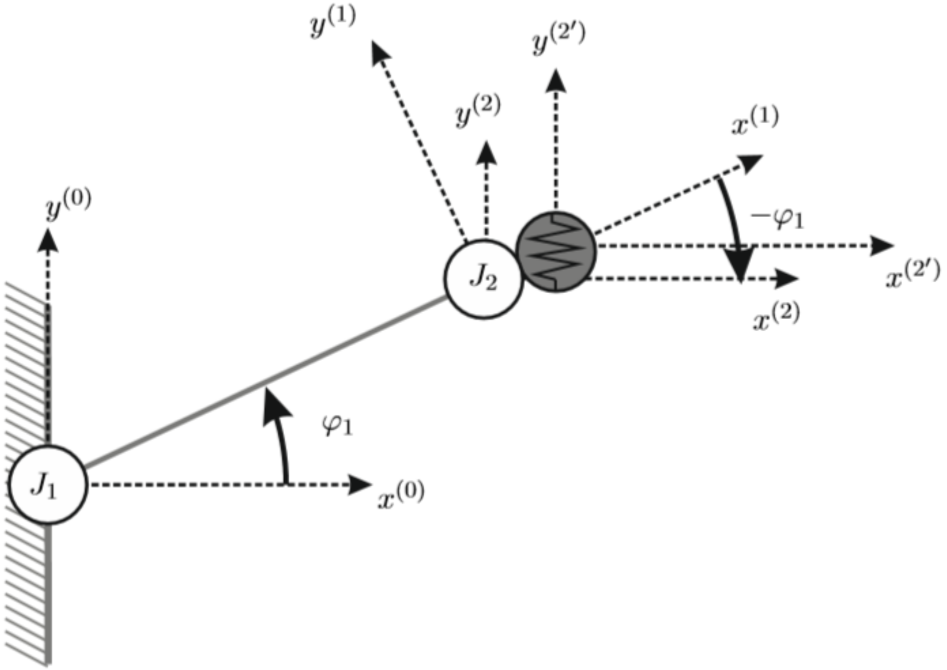
$$\bar{Q}_{UP} = [\bar{\theta}_{11'}^T, \bar{q}_{UP}^T]^T_{8\times1}, \quad \bar{q}_{UP} = [q_{aUP}\ q_{pUP}]^T_{2\times1}, \quad q_{aUP} = [\varphi_{11}]_{1\times1}, \quad q_{pUP} = [\varphi_{12}]_{1\times1}$$

$$\bar{Q}_{DN} = [\bar{\theta}_{21'}^T, \bar{q}_{DN}^T]^T_{8\times1}, \quad \bar{q}_{DN} = [\bar{q}_{pDN}]^T_{2\times1}, \quad \bar{q}_{pDN} = [\varphi_{21}\ \varphi_{22}]^T_{2\times1}$$

$$\begin{bmatrix} (\boldsymbol{J}_{\theta i}\boldsymbol{K}_{\theta i}^{-1}\boldsymbol{J}_{\theta i}^T) & \boldsymbol{J}_{pi} \\ \boldsymbol{J}_{pi}^T & \boldsymbol{0} \end{bmatrix}^{-1} = \left[\begin{array}{c|c} [\boldsymbol{K}_{Ci}]_{6\times6} & \sim \\ \hline \sim & \sim \end{array}\right]$$

$$\boldsymbol{K}_{C(UP)}(\varphi_1) \qquad \boldsymbol{K}_{C(DN)}(\varphi_1)$$

SMY'nin Fazla Kısıtlı Düzlemsel Bir Paralel Mekanizmaya Uygulanması (Aktif Paralelogram)



$$\boldsymbol{K}_{C(active)}(\varphi_1) = \boldsymbol{K}_{C(UP)}(\varphi_1) + \boldsymbol{K}_{C(DN)}(\varphi_1)$$

$$\boldsymbol{K}_{C(Eq)} = \boldsymbol{K}_{C(active)}(\varphi_1)$$

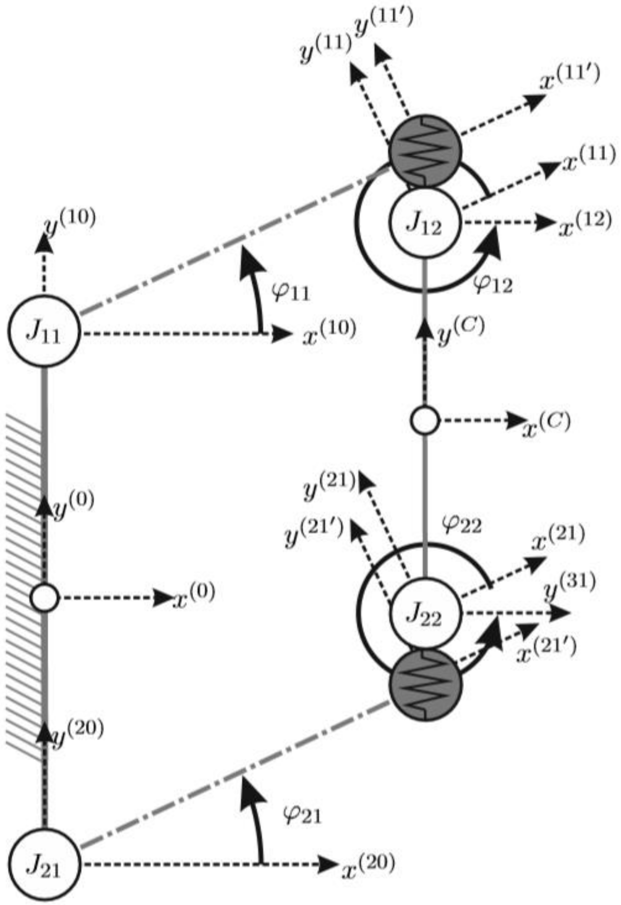
$$\operatorname{rank} \boldsymbol{K}_{C(UP)} = 5 \qquad \operatorname{rank} \boldsymbol{K}_{C(DN)} = 4$$

$$\operatorname{rank} \boldsymbol{K}_{C(active)} = 6$$

$$\boldsymbol{H}_{Eq} = \boldsymbol{R}_3(\varphi_1)\boldsymbol{T}_1(l_1)\boldsymbol{R}_3(-\varphi_1)\boldsymbol{H}_v(\bar{\theta}_{2'})$$

$$\bar{Q}_{Eq} = [\bar{\theta}_{2'}^T \; q_{Eq}]_{7\times 1}^T, \quad q_{Eq} = [q_{aEq}]_{1\times 1}, \quad q_{aEq} = [\varphi_1]_{1\times 1}$$

SMY'nin Fazla Kısıtlı Düzlemsel Bir Paralel Mekanizmaya Uygulanması (Pasif paralelogramlar)



$$\boldsymbol{H}_{UP} = \boldsymbol{T}_2(d/2)\boldsymbol{R}_3(\varphi_{11})\boldsymbol{T}_1(l_1)\boldsymbol{H}_v(\bar{\theta}_{11'})\boldsymbol{R}_3(\varphi_{12})\boldsymbol{T}_2(-d/2)$$
$$\boldsymbol{H}_{DN} = \boldsymbol{T}_2(-d/2)\boldsymbol{R}_3(\varphi_{21})\boldsymbol{T}_1(l_1)\boldsymbol{H}_v(\bar{\theta}_{21'})\boldsymbol{R}_3(\varphi_{22})\boldsymbol{T}_2(d/2)$$
$$\boldsymbol{H}_{UP} = \boldsymbol{H}_{DN}$$

$$\varphi_{11} = \varphi_{21}, \quad \varphi_{12} = \varphi_{22}, \quad \varphi_{11} + \varphi_{12} = \varphi_{21} + \varphi_{22} = 2\pi k \ \text{ for } \ k = 1, 2, \dots, \infty$$

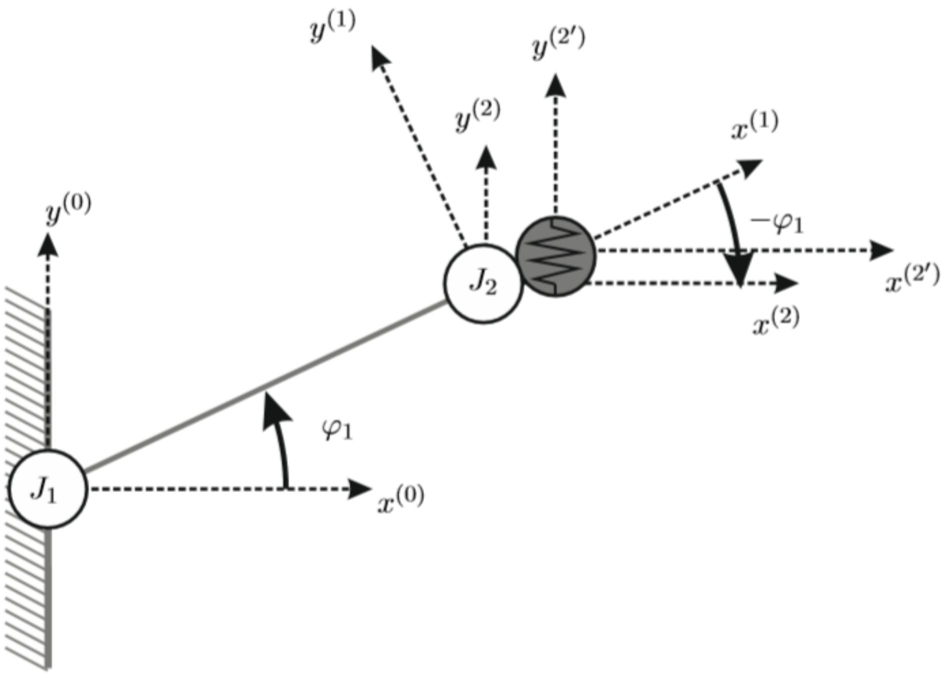
Değişkenler:

$$\bar{Q}_{UP} = [\bar{\theta}_{11'}^T, \bar{q}_{UP}^T]_{8\times1}^T, \quad \bar{q}_{UP} = [\bar{q}_{pUP}]_{2\times1}^T, \quad \bar{q}_{pUP} = [\varphi_{11}\ \varphi_{12}]_{2\times1}^T$$
$$\bar{Q}_{DN} = [\bar{\theta}_{21'}^T, \bar{q}_{DN}^T]_{8\times1}^T, \quad \bar{q}_{DN} = [\bar{q}_{pDN}]_{2\times1}^T, \quad \bar{q}_{pDN} = [\varphi_{21}\ \varphi_{22}]_{2\times1}^T$$

$$\begin{bmatrix} (\boldsymbol{J}_{\theta i}\boldsymbol{K}_{\theta i}^{-1}\boldsymbol{J}_{\theta i}^T) & \boldsymbol{J}_{pi} \\ \boldsymbol{J}_{pi}^T & \boldsymbol{0} \end{bmatrix}^{-1} = \left[\begin{array}{c:c} [\boldsymbol{K}_{Ci}]_{6\times6} & \sim \\ \hdashline \sim & \sim \end{array}\right] \Longrightarrow (\boldsymbol{J}_{\theta i}^{-T}\boldsymbol{K}_{\theta i}\boldsymbol{J}_{\theta i}^{-1})$$

$$\boldsymbol{K}_{C(UP)}(\varphi_1) \qquad \boldsymbol{K}_{C(DN)}(\varphi_1)$$

SMY'nin Fazla Kısıtlı Düzlemsel Bir Paralel Mekanizmaya Uygulanması (Pasif Paralelogram)



$$\boldsymbol{K}_{C(passive)}(\varphi_1) = \boldsymbol{K}_{C(UP)}(\varphi_1) + \boldsymbol{K}_{C(DN)}(\varphi_1)$$

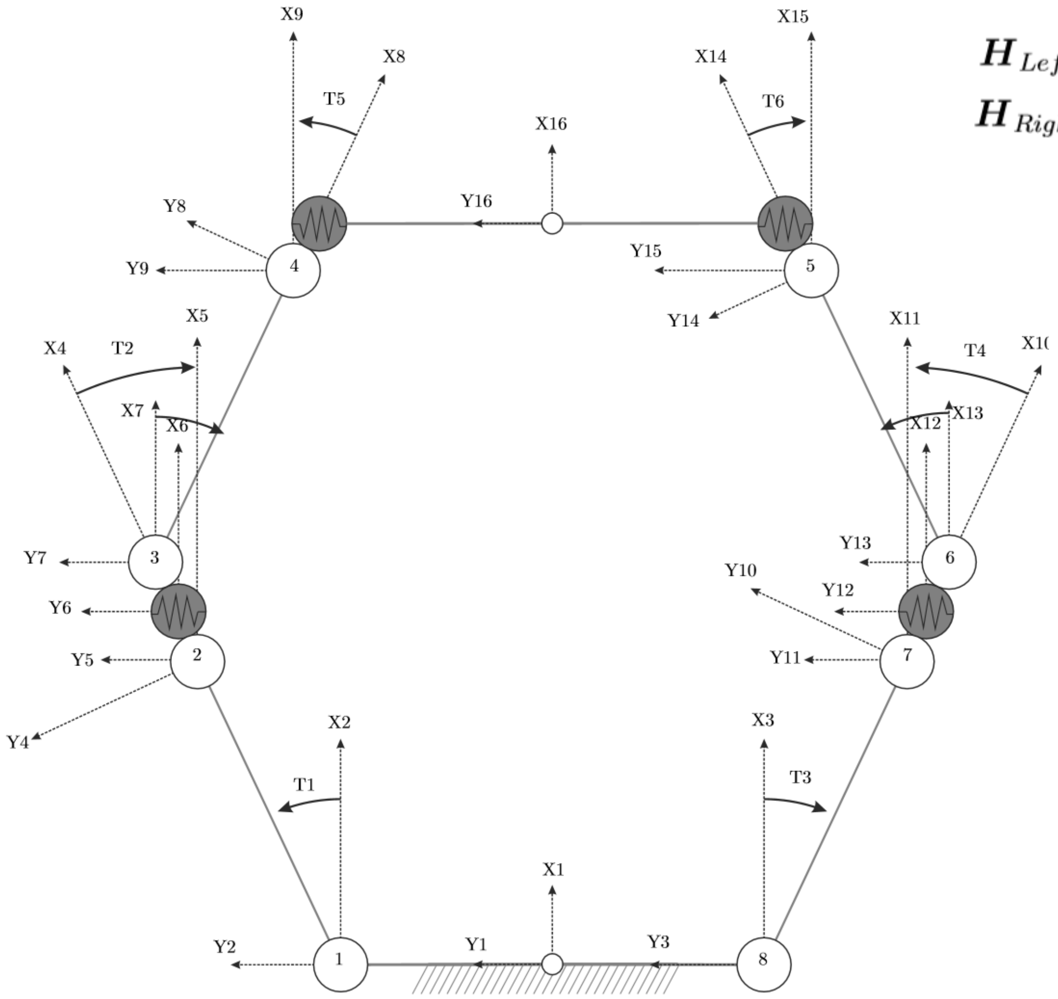
$$\boldsymbol{K}_{C(Eq)} = \boldsymbol{K}_{C(passive)}(\varphi_1)$$

$$\text{rank}\, \boldsymbol{K}_{C(UP)} = \text{rank}\, \boldsymbol{K}_{C(DN)} = 6$$

$$\boldsymbol{H}_{Eq} = \boldsymbol{R}_3(\varphi_1)\boldsymbol{T}_1(l_1)\boldsymbol{R}_3(-\varphi_1)\boldsymbol{H}_v(\bar{\theta}_{2'})$$

$$\bar{Q}_{Eq} = [\bar{\theta}_{2'}^T \; q_{Eq}]_{7\times1}^T, \quad q_{Eq} = [q_{aEq}]_{1\times1}, \quad q_{aEq} = [\varphi_1]_{1\times1}$$

SMY’nin Fazla Kısıtlı Düzlemsel Bir Paralel Mekanizmaya Uygulanması



$$\boldsymbol{H}_{Left} = \boldsymbol{T}_2(d/2)\boldsymbol{R}_3(\varphi_{11})\boldsymbol{T}_1(l_1)\boldsymbol{R}_3(-\varphi_{11})\boldsymbol{H}_v(\bar{\theta}_{11'})\boldsymbol{R}_3(\varphi_{12})\boldsymbol{T}_1(l_1)\boldsymbol{R}_3(-\varphi_{12})\boldsymbol{H}_v(\bar{\theta}_{12'})\boldsymbol{T}_2(-d/2)$$

$$\boldsymbol{H}_{Right} = \boldsymbol{T}_2(-d/2)\boldsymbol{R}_3(\varphi_{21})\boldsymbol{T}_1(l_1)\boldsymbol{R}_3(-\varphi_{21})\boldsymbol{H}_v(\bar{\theta}_{21'})\boldsymbol{R}_3(\varphi_{22})\boldsymbol{T}_1(l_1)\boldsymbol{R}_3(-\varphi_{22})\boldsymbol{H}_v(\bar{\theta}_{22'})\boldsymbol{T}_2(d/2)$$

$$\varphi_{22} = \varphi_{11} = \varphi_1$$

$$\varphi_{12} = \varphi_{21} = \varphi_2$$

$$\bar{Q}_{Left} = \left[\bar{\theta}^T_{Left}\ \bar{q}^T_{Left}\right]^T_{14\times1},\quad \bar{q}_{Left} = \left[q_{aLeft}\ q_{pLeft}\right]^T_{2\times1},\quad q_{aLeft} = \left[\varphi_{11}\right]_{1\times1}$$

$$q_{pLeft} = \left[\varphi_{12}\right]_{1\times1},\quad \bar{\theta}_{Left} = \left[\bar{\theta}^T_{11'},\ \bar{\theta}^T_{12'}\right]^T_{12\times1}$$

$$\bar{Q}_{Right} = \left[\bar{\theta}^T_{Right}\ \bar{q}^T_{Right}\right]^T_{14\times1},\quad \bar{q}_{Right} = \left[q_{aRight}\ q_{pRight}\right]^T_{2\times1},\quad q_{aRight} = \left[\varphi_{21}\right]_{1\times1}$$

$$q_{pRight} = \left[\varphi_{22}\right]_{1\times1},\quad \bar{\theta}_{Right} = \left[\bar{\theta}^T_{21'},\ \bar{\theta}^T_{22'}\right]^T_{12\times1}$$

SMY’nin Fazla Kısıtlı Düzlemsel Bir Paralel Mekanizmaya Uygulanması

$$\boldsymbol{K}_{\theta Left}(\varphi_{11}, \varphi_{12}) = diag\left(\boldsymbol{K}_{C(active)}(\varphi_{11}), \boldsymbol{K}_{C(passive)}(\varphi_{11})\right)$$

$$\boldsymbol{K}_{\theta Right}(\varphi_{21}, \varphi_{22}) = diag\left(\boldsymbol{K}_{C(active)}(\varphi_{21}), \boldsymbol{K}_{C(passive)}(\varphi_{22})\right)$$

$$\begin{bmatrix} \left(\boldsymbol{J}_{\theta i}\boldsymbol{K}_{\theta i}^{-1}\boldsymbol{J}_{\theta i}^{T}\right) & \boldsymbol{J}_{pi} \\ \boldsymbol{J}_{pi}^{T} & \boldsymbol{0} \end{bmatrix}^{-1} = \left[\begin{array}{c:c} [\boldsymbol{K}_{Ci}]_{6\times 6} & \sim \\ \hdashline \sim & \sim \end{array}\right]$$

$$rank(\boldsymbol{K}_{CRight}) = 5$$

$$rank(\boldsymbol{K}_{CLeft}) = 5$$

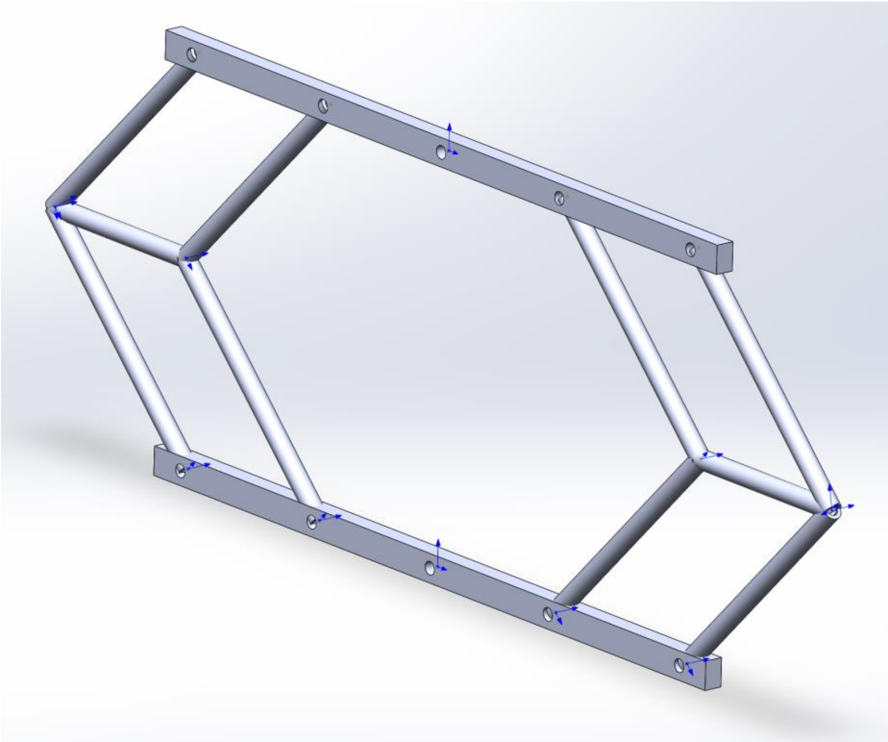
$$\boldsymbol{K}_{CLeft}(\varphi_{11}, \varphi_{12}) \quad \boldsymbol{K}_{CRight}(\varphi_{21}, \varphi_{22})$$

$$\boldsymbol{K}_{C}(\varphi_{11}, \varphi_{12}, \varphi_{21}, \varphi_{22}) = \boldsymbol{K}_{CLeft}(\varphi_{11}, \varphi_{12}) + \boldsymbol{K}_{CRight}(\varphi_{21}, \varphi_{22})$$

$$\begin{array}{l} \varphi_{22} = \varphi_{11} = \varphi_{1} \\ \varphi_{12} = \varphi_{21} = \varphi_{2} \end{array} \Longrightarrow \boldsymbol{K}_{C}(\varphi_{1}, \varphi_{2}) = \boldsymbol{K}_{CLeft}(\varphi_{1}, \varphi_{2}) + \boldsymbol{K}_{CRight}(\varphi_{2}, \varphi_{1})$$

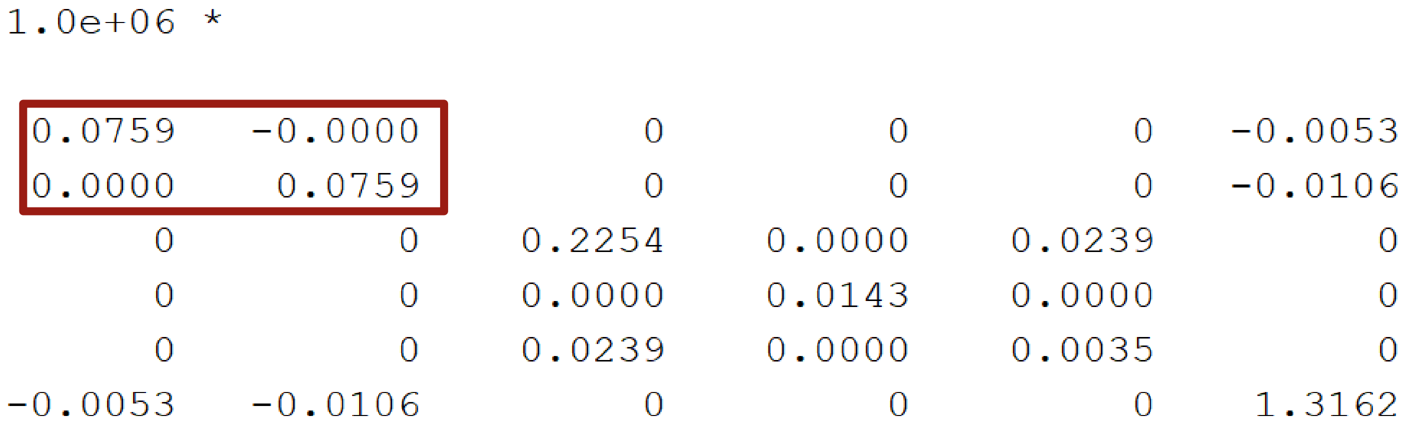
$$rank(\boldsymbol{K}_{C}) = 6$$

SMY’nin Fazla Kısıtlı Düzlemsel Bir Paralel Mekanizmaya Uygulanması



SOLIDWORKS

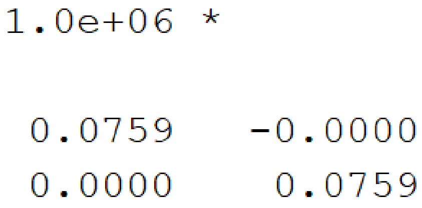
$45^o, -45^o$



```
1.0e+06 *

0.0759   -0.0000         0         0         0   -0.0053
0.0000    0.0759         0         0         0   -0.0106
     0         0    0.2254    0.0000    0.0239         0
     0         0    0.0000    0.0143    0.0000         0
     0         0    0.0239    0.0000    0.0035         0
-0.0053  -0.0106         0         0         0    1.3162
```

$6x6$

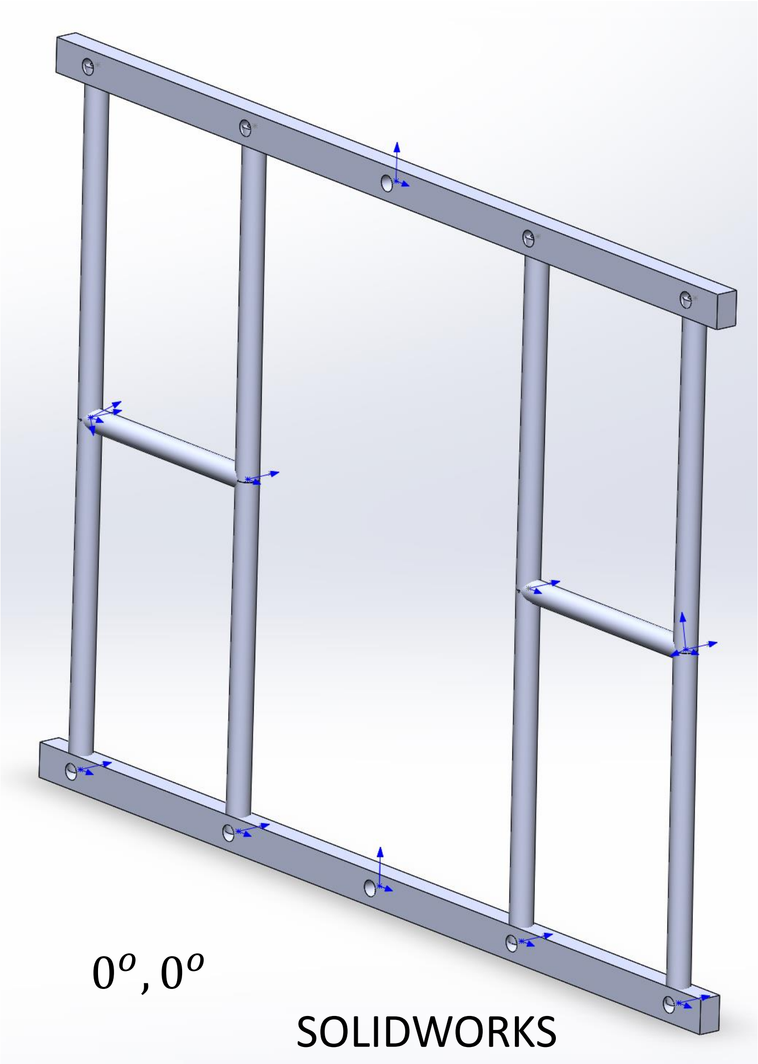


```
1.0e+06 *

0.0759   -0.0000
0.0000    0.0759
```

$2x2$

SMY’nin Fazla Kısıtlı Düzlemsel Bir Paralel Mekanizmaya Uygulanması



```
  1.0e+08 *

    1.3404         0         0         0         0         0
         0         0         0         0         0         0
         0         0    0.0015         0    0.0002         0
         0         0         0    0.0002         0         0
         0         0    0.0002         0    0.0000         0
         0         0         0         0         0    0.0525
```

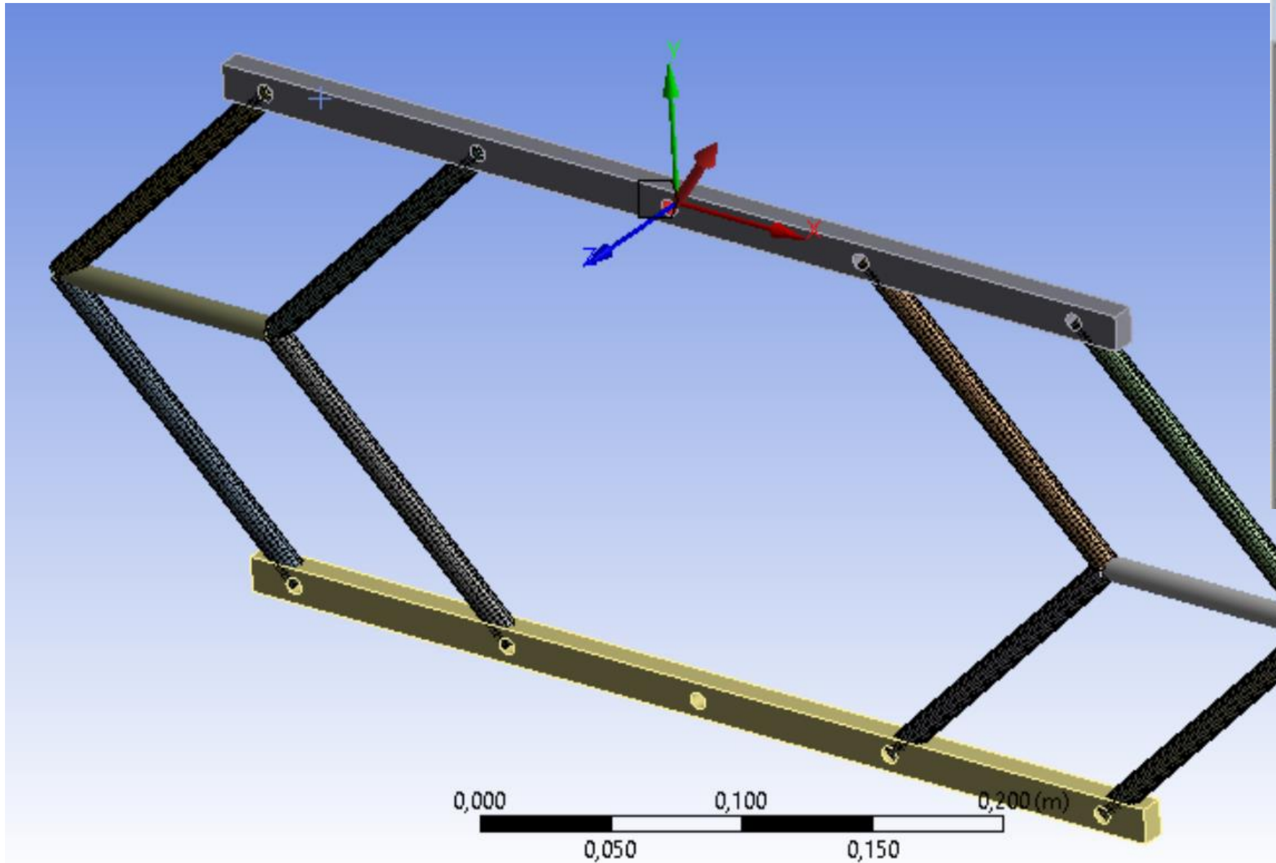
$6x6$

```
  1.0e+08 *

    1.3404         0
         0         0
```

$2x2$

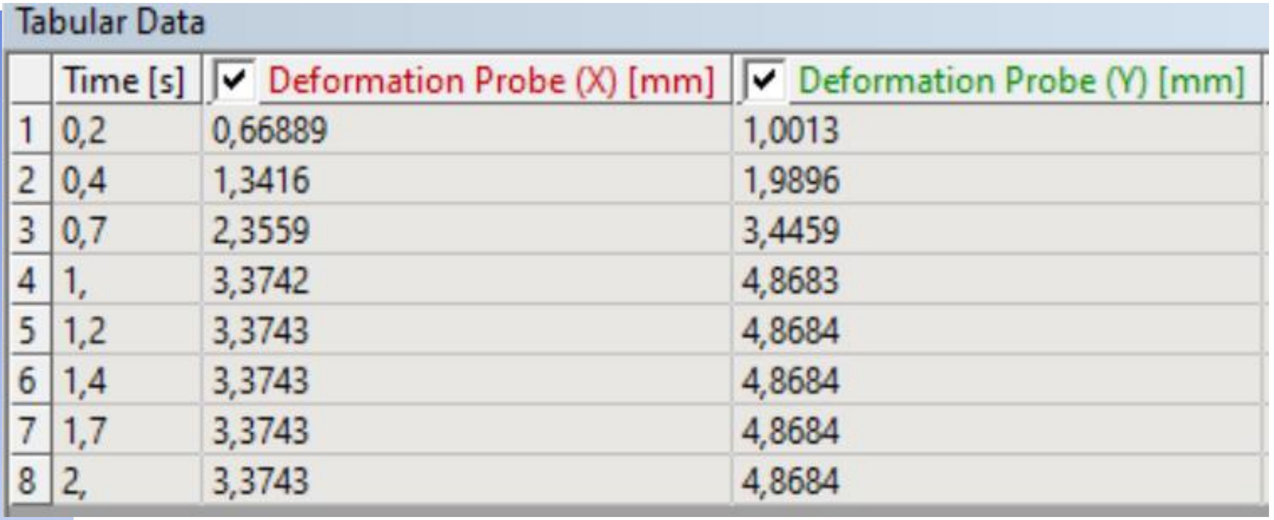
SMY’nin Fazla Kısıtlı Düzlemsel Bir Paralel Mekanizmaya Uygulanması



ANSYS

$45^o, -45^o$

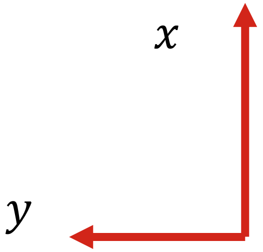
$F = [250; 380; 0]$



Tabular Data

	Time [s]	☑ Deformation Probe (X) [mm]	☑ Deformation Probe (Y) [mm]
1	0,2	0,66889	1,0013
2	0,4	1,3416	1,9896
3	0,7	2,3559	3,4459
4	1,	3,3742	4,8683
5	1,2	3,3743	4,8684
6	1,4	3,3743	4,8684
7	1,7	3,3743	4,8684
8	2,	3,3743	4,8684

```
inv(KC(pi/4,-pi/4))*[380;-250;0;0;0;0]*1000
```

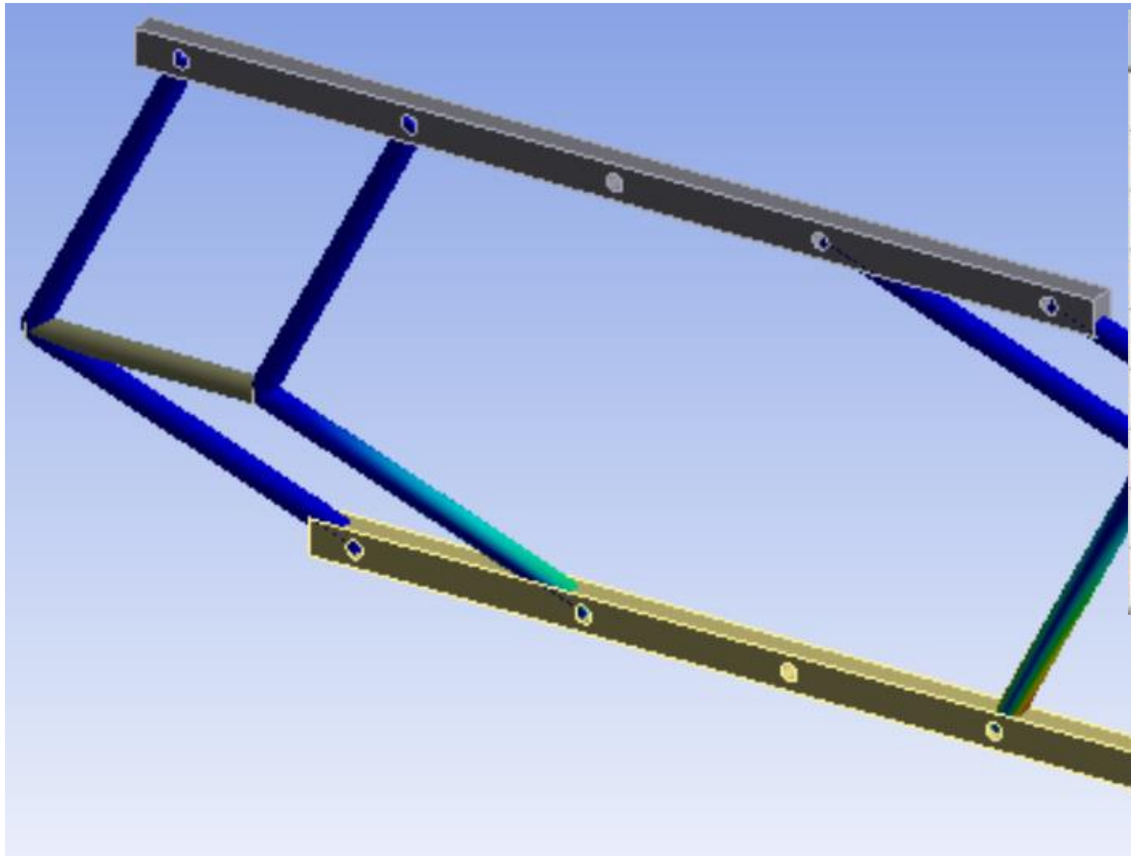


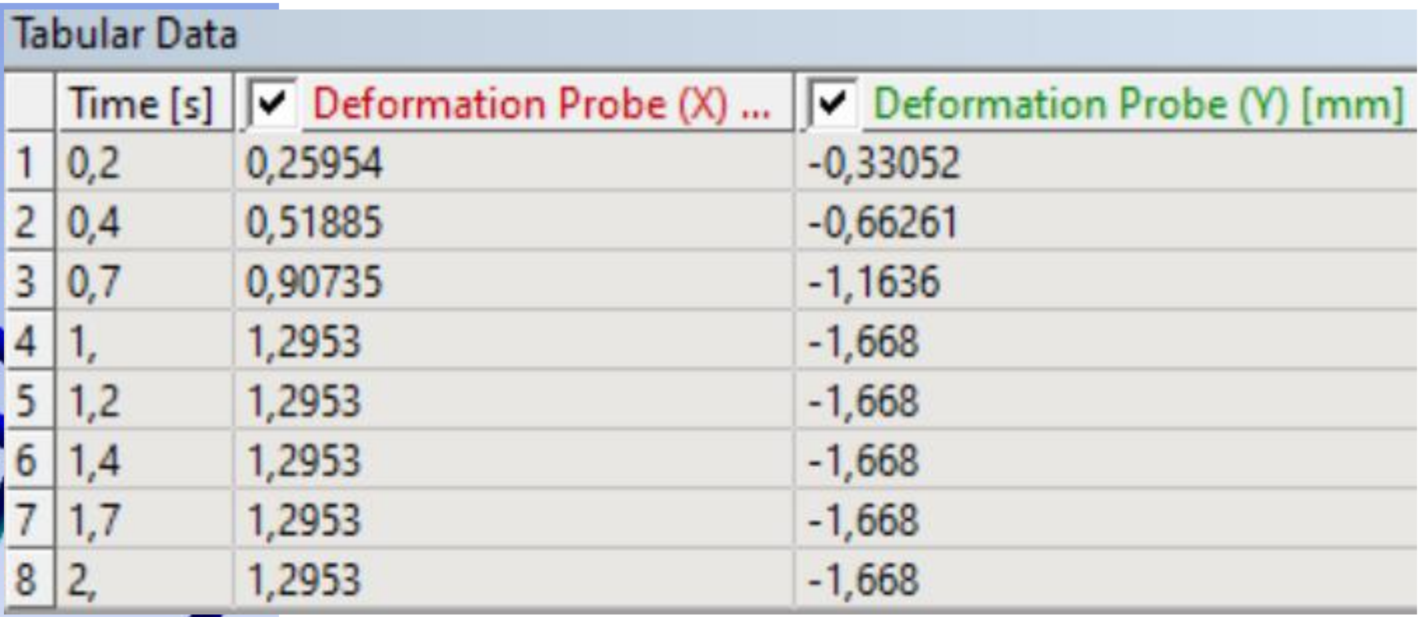
```
ans =

    5.0067
   -3.2950
```

$$F = K\Delta x$$
$$C = K^{-1}$$
$$CF = \Delta x$$

SMY'nin Fazla Kısıtlı Düzlemsel Bir Paralel Mekanizmaya Uygulanması





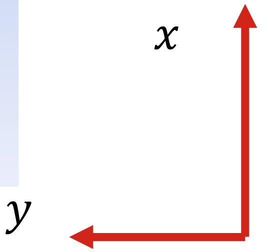
Tabular Data

	Time [s]	☑ Deformation Probe (X) ...	☑ Deformation Probe (Y) [mm]
1	0,2	0,25954	-0,33052
2	0,4	0,51885	-0,66261
3	0,7	0,90735	-1,1636
4	1,	1,2953	-1,668
5	1,2	1,2953	-1,668
6	1,4	1,2953	-1,668
7	1,7	1,2953	-1,668
8	2,	1,2953	-1,668

ANSYS

$80^o, -30^o$

$F = [100; -100; 0]$



```
ans =

   -1.6405
   -1.2900
```

SMY'nin Fazla Kısıtlı Düzlemsel Bir Paralel Mekanizmaya Uygulanması

Dinamik yükler altında esneme.

1. 10 kHz'de çalışabilen direngenlik modeli elde edildi.
2. Çevrimiçi ya da çevrim dışı kullanıma uygun.

SMY’nin Fazla Kısıtlı Düzlemsel Bir Paralel Mekanizmaya Uygulanması

Direngenliğin deneysel belirlenmesi.

1

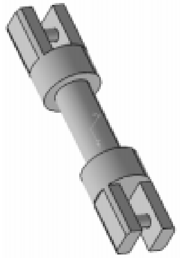
$$\bar{F}_K = \boxed{K_\theta} \Delta\bar{\theta}$$

$$\downarrow$$

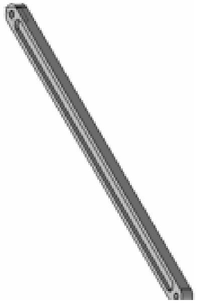
$$(J_\theta^{-T} K_\theta J_\theta^{-1})$$

Uzva yük uygulanır.
Esneme ölçülür.

$$k_{Foot} = \begin{bmatrix} 2.77\cdot10^{-4} & -3.28\cdot10^{-4} & 0 & 0 & 0 & -4.03\cdot10^{-6} \\ -3.28\cdot10^{-4} & 4.14\cdot10^{-4} & 0 & 0 & 0 & 5.41\cdot10^{-6} \\ 0 & 0 & 1.94\cdot10^{-3} & 1.12\cdot10^{-5} & -1.49\cdot10^{-5} & 0 \\ 0 & 0 & 1.12\cdot10^{-5} & 2.29\cdot10^{-7} & 0 & 0 \\ 0 & 0 & -1.49\cdot10^{-5} & 0 & 2.30\cdot10^{-7} & 0 \\ -4.03\cdot10^{-6} & 5.41\cdot10^{-6} & 0 & 0 & 0 & 8.42\cdot10^{-8} \end{bmatrix}$$



$$k_{Axis} = \begin{bmatrix} 6.23\cdot10^{-6} & 0 & 0 & 0 & 0 & 0 \\ 0 & 2.83\cdot10^{-5} & 0 & 0 & 0 & 1.40\cdot10^{-7} \\ 0 & 0 & 2.59\cdot10^{-5} & 0 & -9.65\cdot10^{-7} & 0 \\ 0 & 0 & 0 & 2.77\cdot10^{-7} & 0 & 0 \\ 0 & 0 & -9.65\cdot10^{-7} & 0 & 4.84\cdot10^{-7} & 0 \\ 0 & 1.40\cdot10^{-7} & 0 & 0 & 0 & 1.20\cdot10^{-7} \end{bmatrix}$$



$$k_{Bar} = \begin{bmatrix} 4.55\cdot10^{-5} & 0 & 0 & 0 & 0 & 0 \\ 0 & 2.33\cdot10^{-1} & 0 & 0 & 0 & 1.13\cdot10^{-4} \\ 0 & 0 & 5.08\cdot10^{-2} & 0 & -2.39\cdot10^{-4} & 0 \\ 0 & 0 & 0 & 2.88\cdot10^{-5} & 0 & 0 \\ 0 & 0 & -2.39\cdot10^{-4} & 0 & 1.50\cdot10^{-6} & 0 \\ 0 & 1.13\cdot10^{-3} & 0 & 0 & 0 & 7.19\cdot10^{-6} \end{bmatrix}$$

Klimchik, A. (2011). *Enhanced stiffness modeling of serial and parallel manipulators for robotic-based processing of high performance materials* (Doctoral dissertation).

SMY'nin Fazla Kısıtlı Düzlemsel Bir Paralel Mekanizmaya Uygulanması

Direngenliğin deneysel belirlenmesi.

2

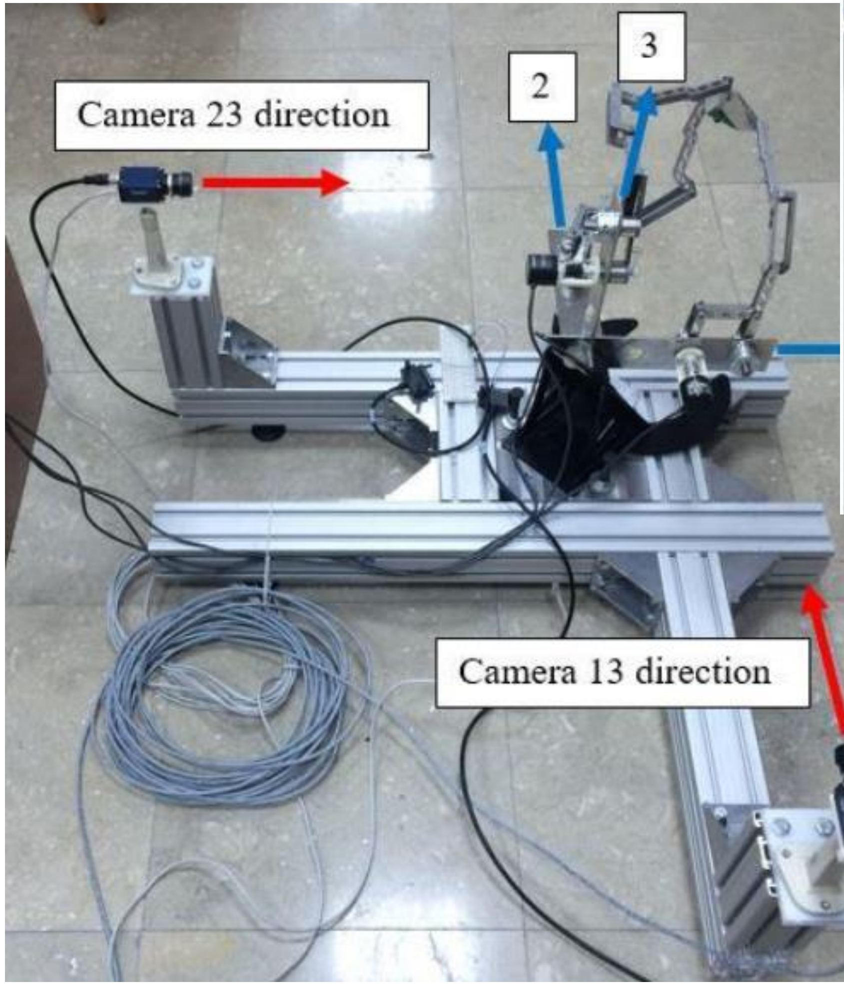
$$\bar{F}_{ext} = K_C \Delta\bar{X}$$

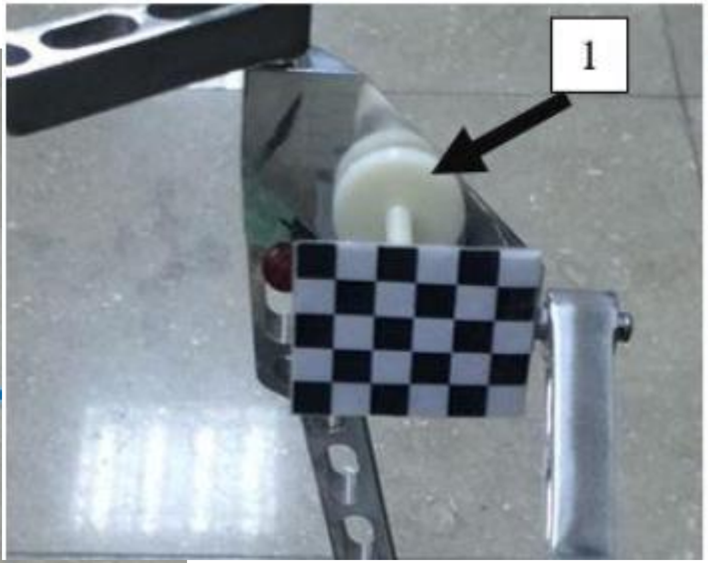
$$\Big\Downarrow$$

$$(J_\theta^{-T} K_\theta J_\theta^{-1})$$

Manipülatöre yük uygulanır. Uç eyleyici esnemesi ölçülür. Uzuv direngenliği hesaplanır.

Görüntü işleme yöntemi

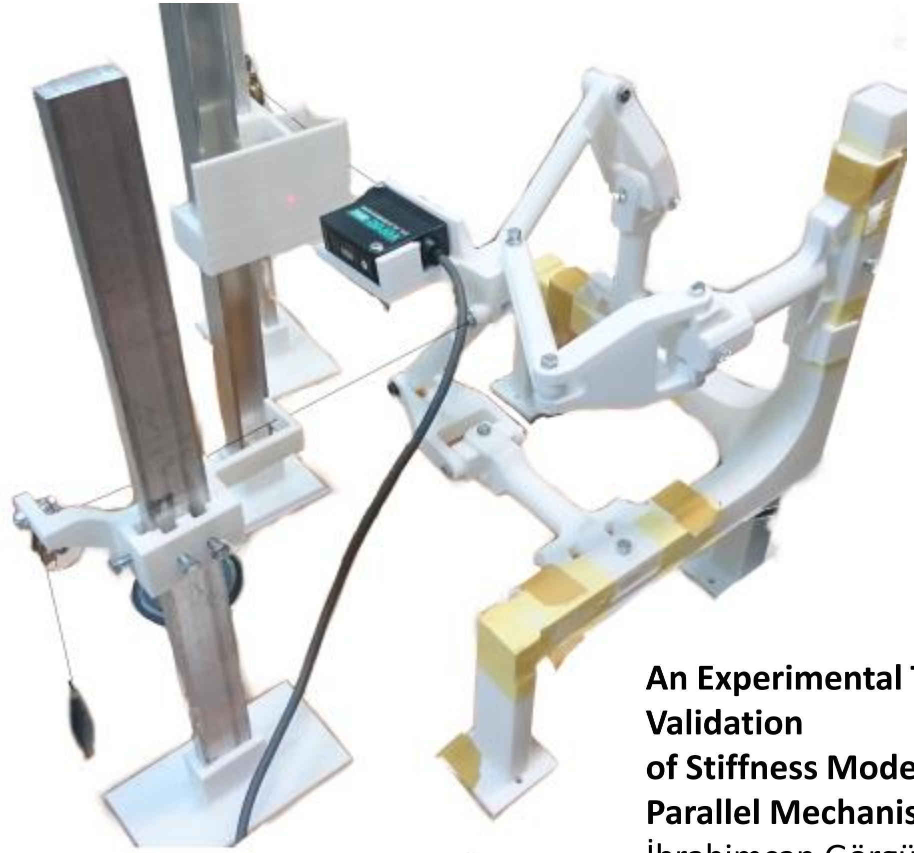




Taner, B., Dede, M.I.C.: Image processing based stiffness mapping of a haptic device. In: New Advances in Mechanisms, Mechanical Transmissions and Robotics, pp. 447{454. Springer (2017)

SMY’nin Fazla Kısıtlı Düzlemsel Bir Paralel Mekanizmaya Uygulanması

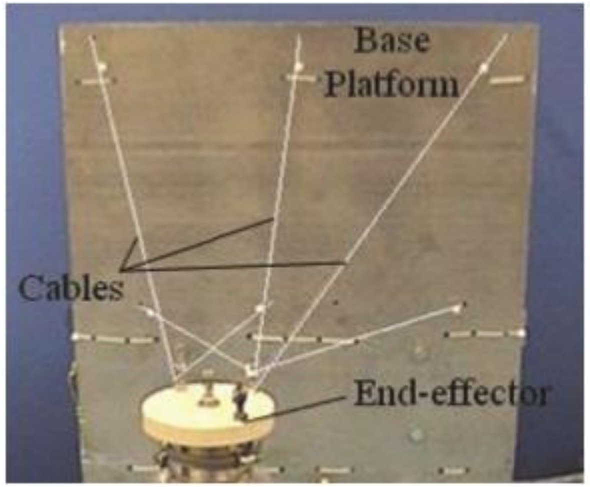
Lazer mesafe sensörü

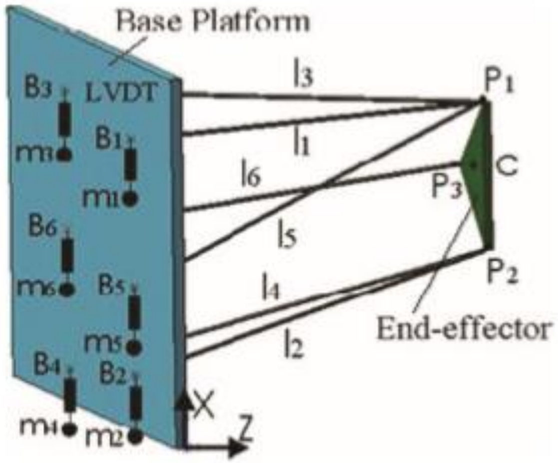


An Experimental Test Procedure for Validation of Stiffness Model: A Case Study for R-CUBE Parallel Mechanism

İbrahimcan Görgülü, M. I. Can Dede, and Giuseppe Carbone

LVDT sensör

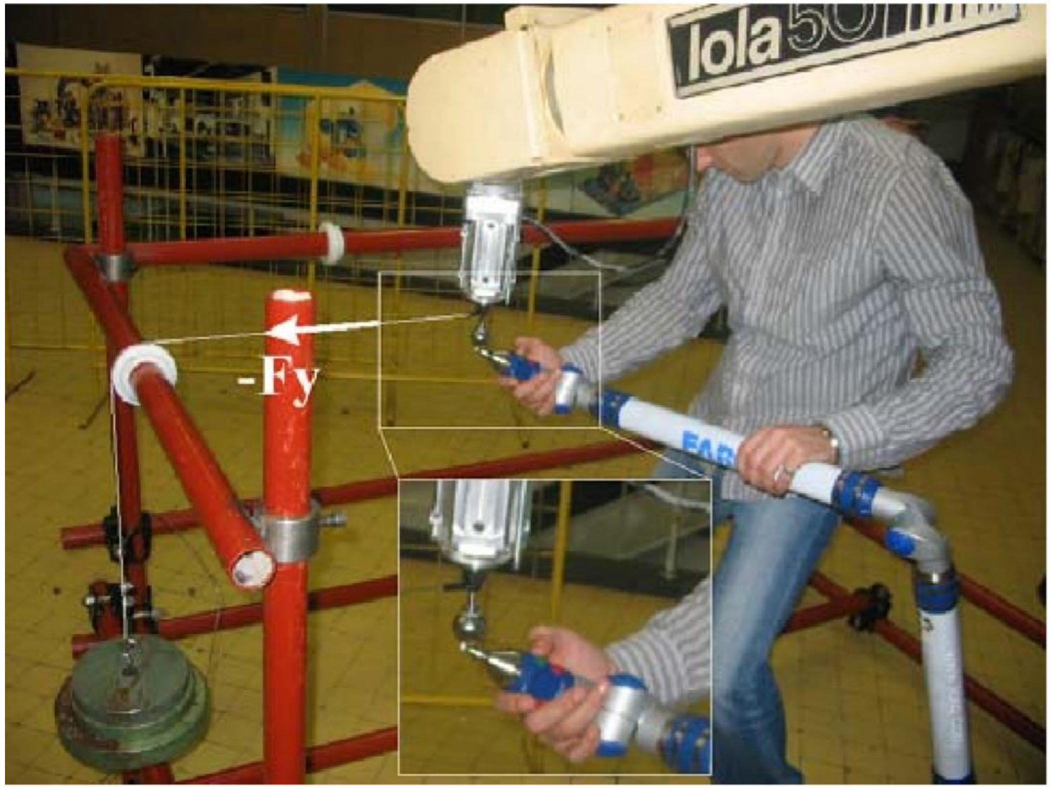




Hernandez-Martinez, E., Carbone, G., & Lopez-Cajun, C. (2009). Operation Features of Milli-CaTraSys. In Proceedings of EUCOMES 08 (pp. 191-199). Springer, Dordrecht.

SMY’nin Fazla Kısıtlı Düzlemsel Bir Paralel Mekanizmaya Uygulanması

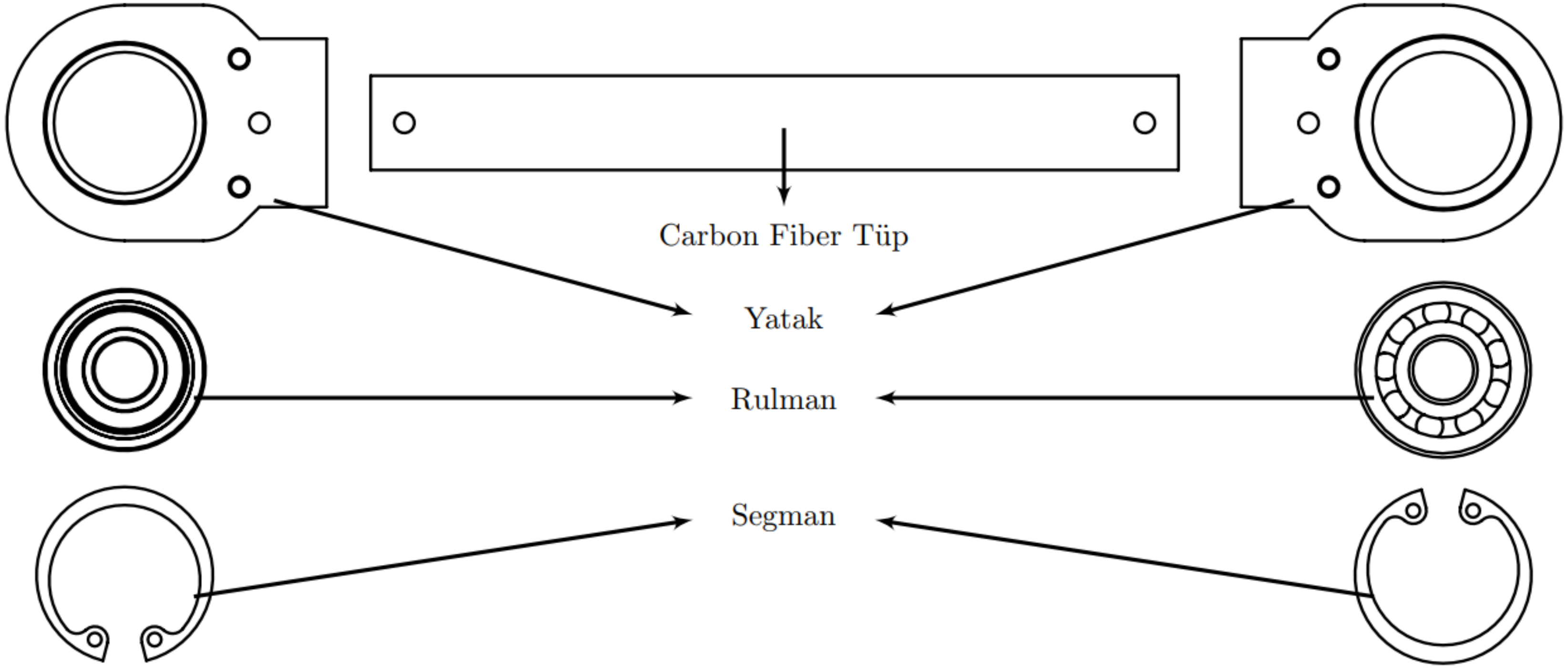
CMM



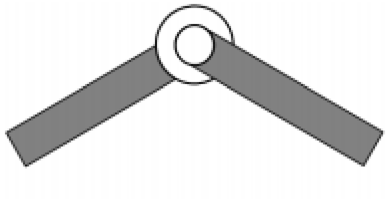
Slavkovi´c, N.R., Milutinovi´c, D.S., Kokotovi´c, B.M., Glavonji´c, M.M., Zivanovi´c, ˇ
S.T., Ehmann, K.F.: Cartesian compliance identification and analysis of an articulated machining robot. FME Transactions **41**(2), 83{95 (2013)

Bonus

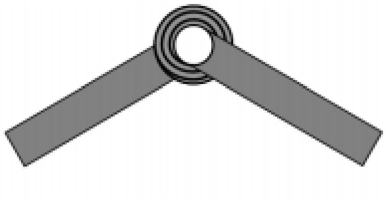
Mafsal direngenliğinin dahil edilmesi.



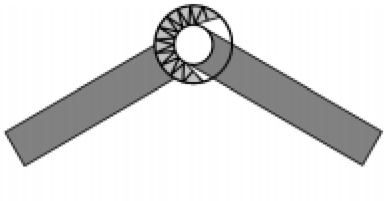
Bonus



Tamamen pasif.

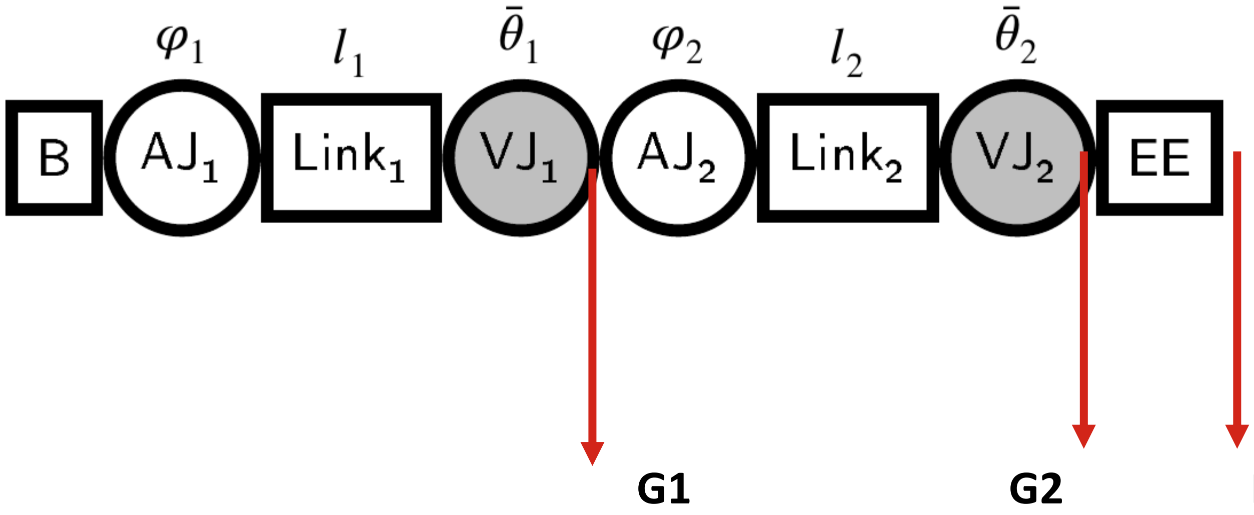


Yay



Lineer olmayan yay.

Statik-Dinamik sürtünmeler. (Kuvvet değişimi)



$$\boldsymbol{J}_\theta^T \bar{F}_{ext} + \boldsymbol{J}_1^T \bar{G}_1 + \boldsymbol{J}_2^T \bar{G}_2 = \boldsymbol{K}_\theta \boldsymbol{J}_\theta^{-1} \Delta \bar{X}$$

$$\boldsymbol{J}_\theta \boldsymbol{K}_\theta^{-1} [\boldsymbol{J}_\theta^T \boldsymbol{J}_1^T \boldsymbol{J}_2^T][\bar{F}_{ext}^T \bar{G}_1^T \bar{G}_2^T]^T = \Delta \bar{X}$$

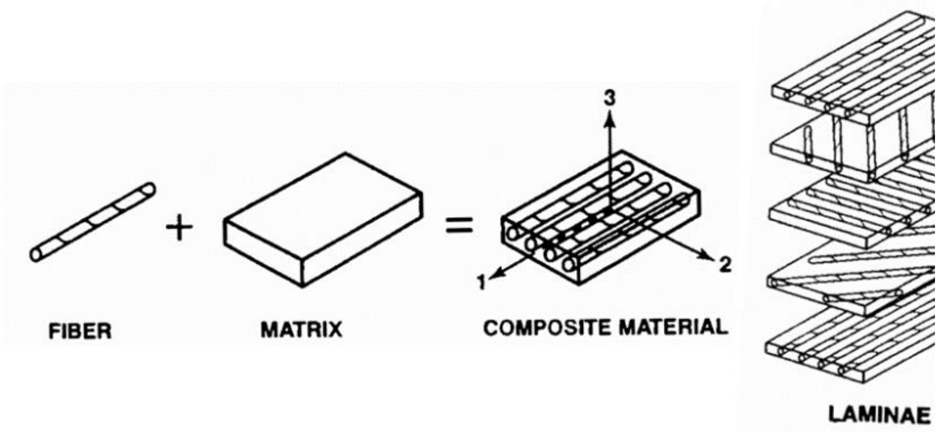
Bonus

Montaj sonrası iç gerilmeler

$$\boldsymbol{K}_{\theta 2}\boldsymbol{J}_{\theta 2}^{-1}\Delta\bar{X}_2 = \boldsymbol{K}_{\theta 1}\boldsymbol{J}_{\theta 1}^{-1}\Delta\bar{X}_1$$

$$\bar{F}_1 = \bar{F}_2$$

$$\boldsymbol{J}_{\theta 1}^{T}\bar{F}_1 = \boldsymbol{J}_{\theta 2}^{T}\bar{F}_2$$



$$\hat{K}_{\theta_k} = \begin{bmatrix} \frac{E_{xk}A_k}{L_k} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{12E_{yk}I_{zk}}{L_k^3} & 0 & 0 & 0 & \frac{-6E_{zk}I_{zk}}{L_k^2} \\ 0 & 0 & \frac{12E_{zk}I_{yk}}{L_k^3} & 0 & \frac{6E_{yk}I_{yk}}{L_k^2} & 0 \\ 0 & 0 & 0 & \frac{G_{xyk}J_k}{L_k} & 0 & 0 \\ 0 & 0 & \frac{6E_{yk}I_{yk}}{L_k^2} & 0 & \frac{4E_{yk}I_{yk}}{L_k} & 0 \\ 0 & \frac{-6E_{zk}I_{zk}}{L_k^2} & 0 & 0 & 0 & \frac{4E_{zk}I_{zk}}{L_k} \end{bmatrix}$$

İlginiz için teşekkürler…

Sorular?