KINEMATIC ANALYSIS OF PARALLEL MANIPULATORS

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DEFINITION OF A PARALLEL MANIPULATOR

A parallel manipulator is a multi-DoF mechanical system that has at least *two separate kinematic chains* between *the end-effector* and *the base*.

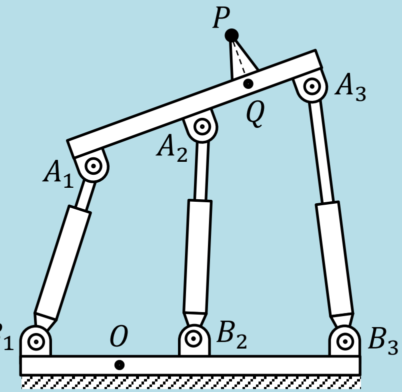
The link carrying the end-effector and the link attached to the base are conceived as *platforms*.

They are called *terminal platform* and *fixed platform*.

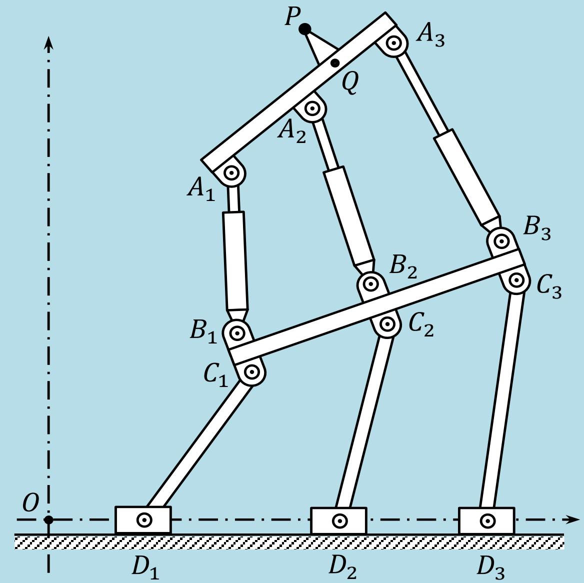
The terminal platform is also called *moving platform* if the parallel manipulator has only two platforms.

In general, a parallel manipulator may also contain *intermediate platforms*, which are inserted in order to increase the mobility of the manipulator and the reachability range of the end-effector.

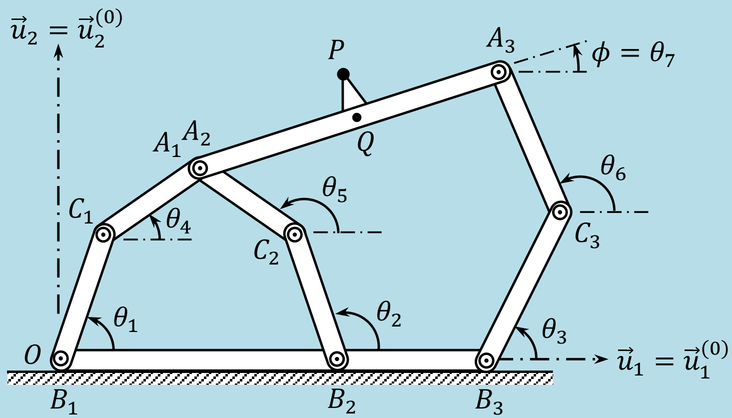
Examples of Planar Parallel Manipulators



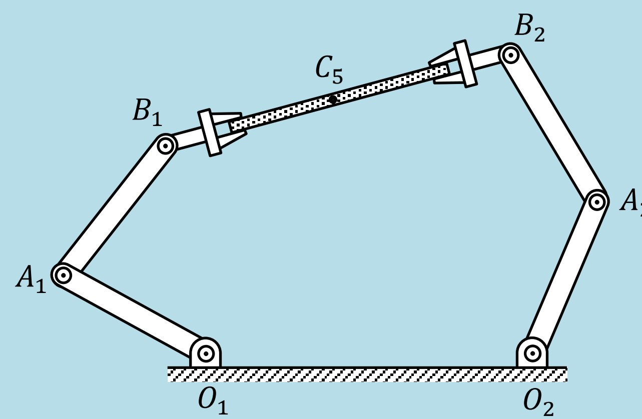
3RPR Manipulator



3PRR+3RPR Manipulator



3RRR Manipulator



2RRR Manipulator

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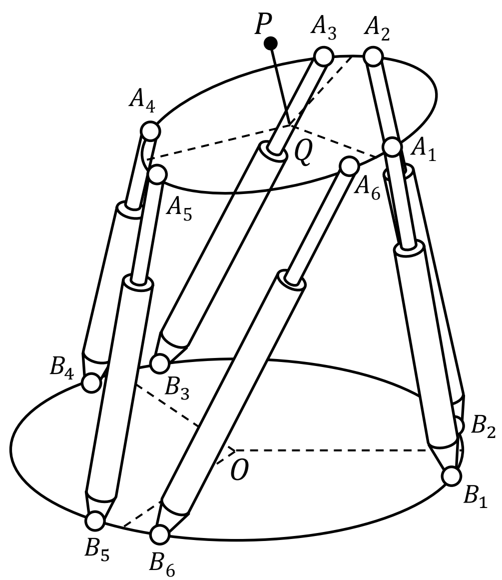
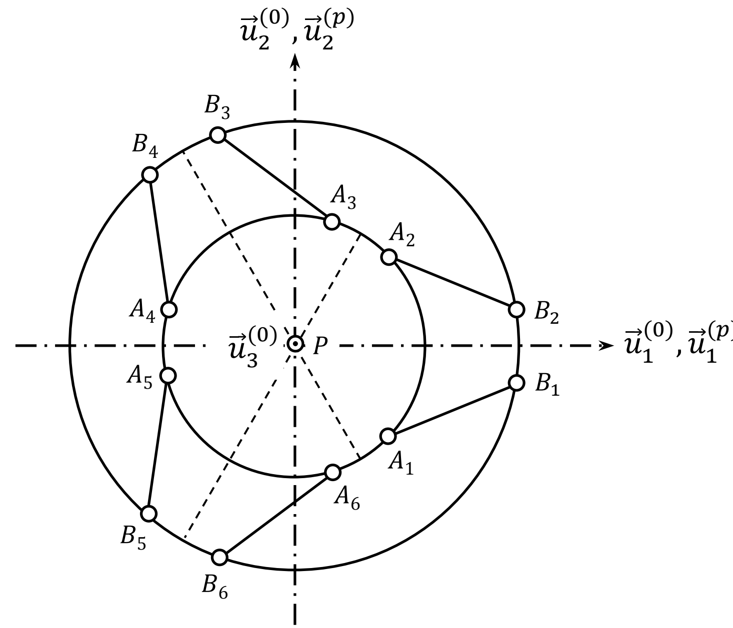
Examples of Spatial Parallel Manipulators

Stewart-Gough Platform (6UPS Parallel Manipulator)



General View



Top View in the Parking Pose

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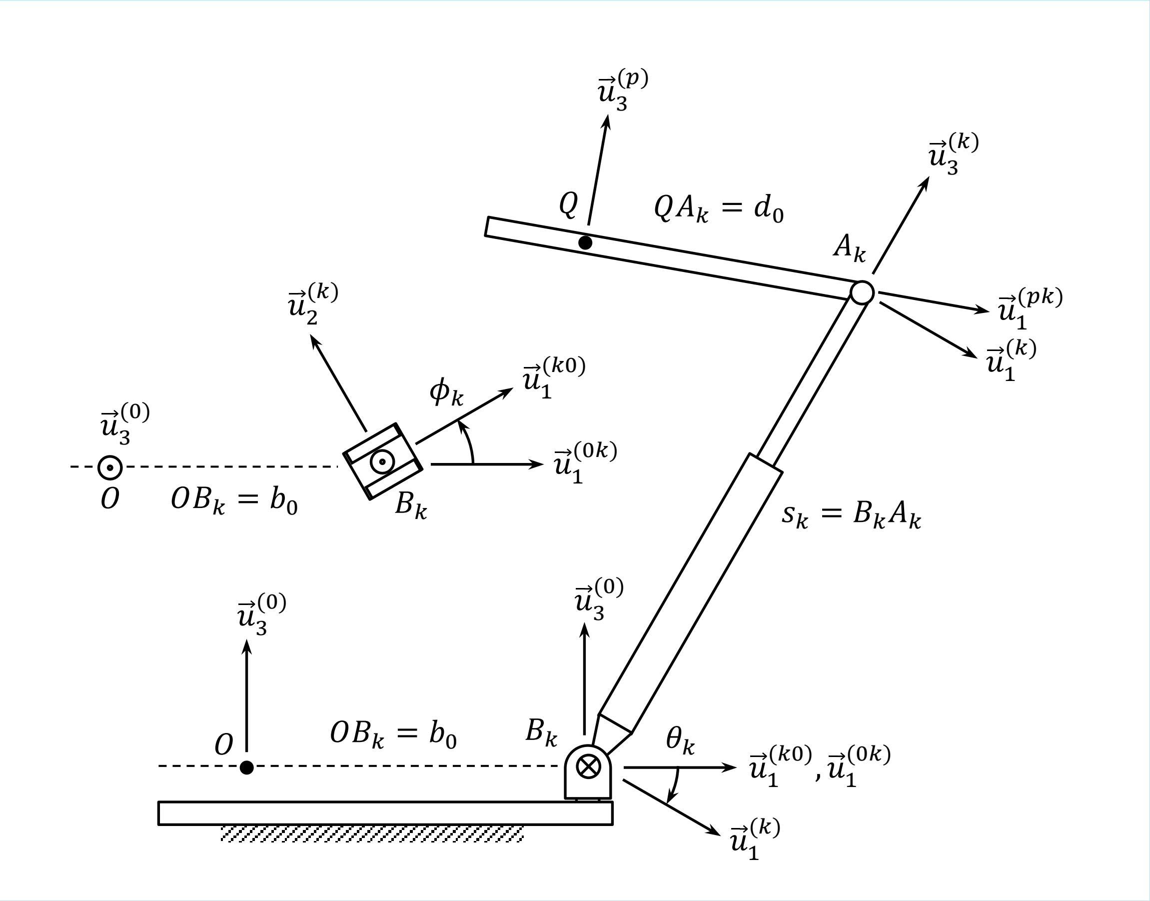
Examples of Spatial Parallel Manipulators

Stewart-Gough Platform (6UPS Parallel Manipulator)



The Details of the Leg L_k

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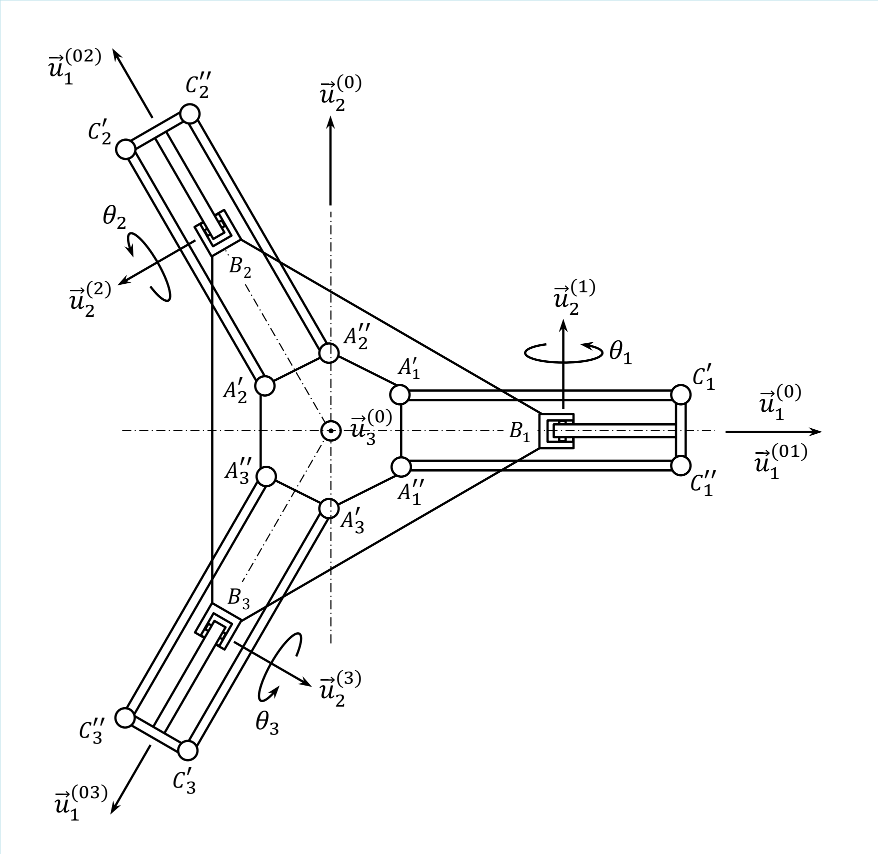
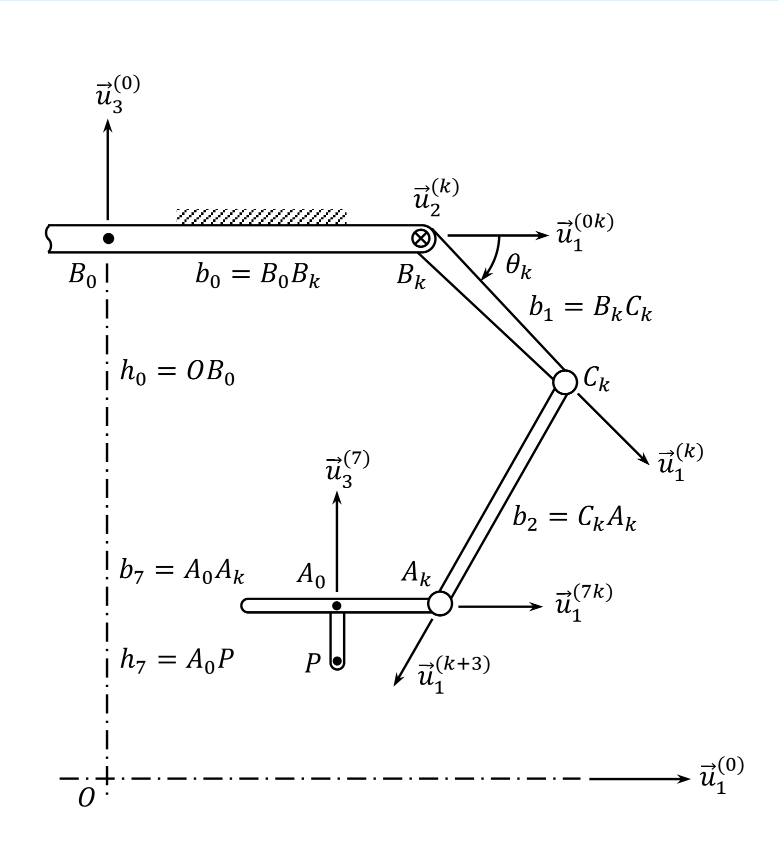
Examples of Spatial Parallel Manipulators

Delta Robot ($3RS^2S^2$ Parallel Manipulator)



Top View in the Parking Pose



The Details of the Leg L_k

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Kinematic Characteristics of a Regular Spatial Parallel Manipulator

Degree of Freedom (DoF)

Kutzbach-Grübler Formula:

$$\mathrm{DoF} = m = 6n_{mb} - (5j_1 + 4j_2 + 3j_3)$$

$\lambda = 6 = \mathrm{DoF\ of\ the\ Task\ Space}$

$n_{mb} = \mathrm{Number\ of\ the\ Movable\ Bodies\ (i.e.\ Links)}$

$j_k = \mathrm{Number\ of\ the\ Joints\ with}\ k\ \mathrm{Degrees\ of\ Relative\ Freedom}$

Note:

For a *regular manipulator*, the number of actuated joints is equal to $\boldsymbol{\lambda}$.

For a *redundant manipulator*, the number of actuated joints is larger than $\boldsymbol{\lambda}$.

For a *deficient manipu*lator, the number of actuated joints is less than $\boldsymbol{\lambda}$.

Kinematic Characteristics of a Regular Spatial Parallel Manipulator

Number of Independent Kinematic Loops (IKLs)

$$n_{ikl} = j_{tot} - n_{mb} = j_1 + j_2 + j_3 - n_{mb}$$

Number of Primary and Secondary Variables

$$n_{pv} = m = \mathrm{DoF}$$
$$n_{sv} = n = \lambda n_{ikl} = 6n_{ikl}$$

Primary Variables (Generalized Coordinates) $(x_1, x_2, x_3, \dots, x_m)$

They are *necessary* and *sufficient* to describe the position of the whole manipulator completely.

Secondary Variables $(y_1, y_2, y_3, \dots, y_n)$

They are the *necessary* and *sufficient* supplements to the primary variables in order to describe the relative positions of the bodies (i.e. links) completely within the *Independent Kinematic Loops*.

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Kinematic Characteristics of a Regular Spatial Parallel Manipulator

Position (Location and Orientation) of the End-Effector Through the Legs

$\boldsymbol{\rho} = \boldsymbol{f_k}(\boldsymbol{x}, \boldsymbol{y})$ for $k = 1, 2, \ldots, l$

$l =$ Number of the Legs

Leg (or Limb): A kinematic chain between the base and the moving platform.

$\boldsymbol{\rho} \in \mathcal{R}^6$, $\boldsymbol{x} \in \mathcal{R}^m = \mathcal{R}^6$, $\boldsymbol{y} \in \mathcal{R}^n$

$\boldsymbol{\rho} = \begin{bmatrix} \boldsymbol{p} \\ \boldsymbol{\varphi} \end{bmatrix}$: Indicates the *Location* and *Orientation* of the End-Effector

$\boldsymbol{p} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix}$: Consists of the *Tip Point Coordinates*

$\boldsymbol{\varphi} = \begin{bmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{bmatrix}$: Consists of the *Orientation Parameters* of the End-Effector

(e.g. the *Euler Angles* of the 1-2-3 Sequence)

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Kinematic Characteristics of a Regular Spatial Parallel Manipulator

Independent Kinematic Loops (IKLs) formed by the Leg Pairs

For a parallel manipulator with

two platforms (i.e. moving and fixed platforms)

and

serial-chain legs,

the IKLs can be formed based on the *Leg Pairs* taken as

$$\{L_1, L_2\}, \{L_1, L_3\}, \dots, \{L_1, L_l\}$$

Thus, it happens that

$$n_{ikl} = l - 1$$

For example, for the *Stewart-Gough platform* with $l = 6$,

$$n_{ikl} = 6 - 1 = 5$$

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Kinematic Characteristics of a Regular Spatial Parallel Manipulator

Closure Equations for the IKLs

For a parallel manipulator with two platforms and serial-chain legs, the *loop closure equations* can be generated from the *end-effector position equations* written through the legs. That is,

$$\boldsymbol{\phi}(\boldsymbol{x},\boldsymbol{y}) = \begin{bmatrix} \boldsymbol{\phi}_1(\boldsymbol{x},\boldsymbol{y}) \\ \boldsymbol{\phi}_2(\boldsymbol{x},\boldsymbol{y}) \\ \vdots \\ \boldsymbol{\phi}_{l-1}(\boldsymbol{x},\boldsymbol{y}) \end{bmatrix} = \begin{bmatrix} \boldsymbol{f}_2(\boldsymbol{x},\boldsymbol{y}) - \boldsymbol{f}_1(\boldsymbol{x},\boldsymbol{y}) \\ \boldsymbol{f}_3(\boldsymbol{x},\boldsymbol{y}) - \boldsymbol{f}_1(\boldsymbol{x},\boldsymbol{y}) \\ \vdots \\ \boldsymbol{f}_l(\boldsymbol{x},\boldsymbol{y}) - \boldsymbol{f}_1(\boldsymbol{x},\boldsymbol{y}) \end{bmatrix} = \mathbf{0}$$

Note that

$\boldsymbol{\phi}(\boldsymbol{x},\boldsymbol{y}) \in \mathcal{R}^n$ just like $\boldsymbol{y} \in \mathcal{R}^n$

Kinematic Analysis in the Position Domain

Forward Kinematics (Forward Position Analysis)

It involves:

- Forward Kinematic Solution
- Identification and Analysis of PMFKs
- Identification and Analysis of PSFKs

Inverse Kinematics (Inverse Position Analysis)

It involves:

- Inverse Kinematic Solution
- Identification and Analysis of PMIKs
- Identification and Analysis of PSIKs

Acronyms:

PMFK: Posture Mode of Forward Kinematics

PMIK: Posture Mode of Inverse Kinematics

PSFK: Position Singularity of Forward Kinematics

PSIK: Position Singularity of Inverse Kinematics

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Kinematic Analysis in the Velocity Domain

Forward Velocity Analysis

It involves:

- Forward Velocity Solution
- Identification and Analysis of MSFKs

Inverse Velocity Analysis

It involves:

- Inverse Velocity Solution
- Identification and Analysis of MSIKs

Acronyms:

MSFK: Motion Singularity of Forward Kinematics

MSIK: Motion Singularity of Inverse Kinematics

Forward Kinematics

Forward Kinematic Solution

Given: Primary Variables (i.e. $\boldsymbol{x}$)

Find: Position of the End-Effector (i.e. $\boldsymbol{\rho}$)
By-Product: Secondary Variables (i.e. $\boldsymbol{y}$)

Stage 1

Find $\boldsymbol{y}$ in terms of $\boldsymbol{x}$ from the *closure equations* of the IKLs. That is, solve the following equation for $\boldsymbol{y}$.

$$\boldsymbol{\phi}(\boldsymbol{x}, \boldsymbol{y}) = \boldsymbol{0}$$

The solution will be:

$$\boldsymbol{y} = \boldsymbol{h}_{\boldsymbol{i}}(\boldsymbol{x}) \text{ for } i = 1, 2, \ldots, n_{PMFK}$$

Here,

$$n_{PMFK} = \text{Number of PMFKs}$$

Stage 2

Find $\boldsymbol{\rho}$ from one of the *leg-through equations* in terms of $\boldsymbol{x}$ and the selected one of the PMFKs.
If the leg L_k is selected for this purpose, then $\boldsymbol{\rho}$ will be:

$$\boldsymbol{\rho} = \boldsymbol{g}_{\boldsymbol{i}}(\boldsymbol{x}) = \boldsymbol{f}_{\boldsymbol{k}}(\boldsymbol{x}, \boldsymbol{h}_{\boldsymbol{i}}(\boldsymbol{x})) \text{ for } i = 1,2, \ldots, n_{PMFK}$$

Note

The above formula for $\boldsymbol{\rho}$ is *indifferent to the selected leg*, because every leg-through equation is written so that it leads to the same end-effector position.

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Forward Kinematics

PMFKs (Posture Modes of Forward Kinematics)

They are also called *Assembly Modes* of the manipulator.

A PMFK (e.g. PMFK-1) is selected as the manipulator is assembled. Afterwards, it does not change during the operation of the manipulator, unless the manipulator gets into a PMCP (Posture Mode Changing Pose). In a PMCP, the selected PMFK-1 may easily be changed into a different mode (e.g. PMFK-2) without disassembling and reassembling the manipulator.

Therefore, PMCPs must be avoided. Their close vicinities must also be avoided in order to prevent a *loss of accuracy* during the operation.

PSFKs (Position Singularities of Forward Kinematics)

In a pose of PSFK, a certain part of $\boldsymbol{x}$ becomes constrained and therefore cannot be specified as desired. That is, the parts $\boldsymbol{x_b}$ and $\boldsymbol{x_a}$ of $\boldsymbol{x}$ become tied up with a *singularity relationship* such as

$$\boldsymbol{x_b} = \boldsymbol{\xi}(\boldsymbol{x_a})$$

Consequently, a corresponding part $\boldsymbol{y_b}$ of $\boldsymbol{y}$ becomes indefinite, i.e. it cannot be found from the closure equations of the IKLs. This indefiniteness is transmitted to the position of the end-effector, too. That is, a part $\boldsymbol{\rho_b}$ of $\boldsymbol{\rho}$ also becomes indefinite, i.e. uncontrollable by the actuators of the manipulator.

Therefore, the poses of PSFK must be avoided.

They can be avoided in one of the following two ways:

(1) The geometric design parameters are selected so that the singularity relationship never occurs.

(2) The actuators are controlled so that the singularity relationship is never allowed to occur.

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Inverse Kinematics

Inverse Kinematic Solution

Given: Position of the End-Effector (i.e. $\boldsymbol{\rho}$)

Find: Both Primary and Secondary Variables (i.e. $\boldsymbol{x}$ and $\boldsymbol{y}$)

Solution Method

The solution is obtained *leg by leg* from the *leg-through position equations*, which are written as

$$\boldsymbol{f_k}(\boldsymbol{z_k}) = \boldsymbol{\rho} \quad \text{for } k = 1, 2, \dots, l$$

Here,

$$\boldsymbol{z} = \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \end{bmatrix} \in \mathcal{R}^{m+n}$$

and

$\boldsymbol{z_k}$: kth partition of $\boldsymbol{z}$ that belongs to the leg L_k

Note that

$$\boldsymbol{z_k} = \begin{bmatrix} \boldsymbol{x_k} \\ \boldsymbol{y_k} \end{bmatrix} \in \mathcal{R}^m = \mathcal{R}^6$$

Therefore, $\boldsymbol{z_k}$ can be found from the kth leg-through equation.

Hence, $\boldsymbol{x_k}$ and $\boldsymbol{y_k}$ also become available.

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Inverse Kinematics

Solution of the *k*th Leg-Through Equation

$$\boldsymbol{z_k} = \boldsymbol{\psi_{ki}}(\boldsymbol{\rho}) \text{ for } i = 1, 2, \dots, n_{PMIK(k)}$$

Here,

$n_{PMIK(k)}$ = Number of PMIKs for the leg L_k

PMIKs (Posture Modes of Inverse Kinematics)

They are also called *Assembly Modes* of the legs of the manipulator.

A PMIK(*k*), e.g. PMIK(*k*,1), is selected as the leg L_k is assembled. Afterwards, it does not change during the operation of the manipulator, unless L_k gets into a LPMCP(*k*) (Posture Mode Changing Pose of L_k). In an LPMCP(*k*), the selected PMIK(*k*,1) may easily be changed into a different mode, e.g. PMIK(*k*,2), without disassembling and reassembling the leg L_k.

However, LPMCPs need not be avoided, if the legs are *evenly actuated*. Because, in such a manipulator, each leg contains at least one actuator. Therefore, all the legs are under control and thus an undesirable change in the posture mode can be prevented.

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Inverse Kinematics

PSIKs (Position Singularities of Inverse Kinematics)

Since the inverse kinematic solution is obtained leg by leg, a leg L_k may get into a pose PSIK, i.e. PSIK(*k*), *independently* of the other legs. Any two legs, e.g. L_i and L_j, may get into poses of PSIK(*i*) and PSIK(*j*) either simultaneously or at different times.

In a pose of PSIK(*k*), a certain part of $\boldsymbol{\rho}$ becomes constrained and therefore cannot be specified as desired. That is, the parts $\boldsymbol{\rho_{bk}}$ and $\boldsymbol{\rho_{ak}}$ of $\boldsymbol{\rho}$ become tied up with a *singularity relationship* such as

$$\boldsymbol{\rho_{bk}} = \boldsymbol{\xi_k}(\boldsymbol{\rho_{ak}})$$

Consequently, a corresponding part $\boldsymbol{z_{bk}}$ of $\boldsymbol{z_k}$ becomes indefinite, i.e. it cannot be found from the leg-through equation of the leg L_k and it can be specified arbitrarily as desired in addition to the freely specifiable part $\boldsymbol{\rho_{ak}}$ of $\boldsymbol{\rho}$.

On the other hand, the complementary part $\boldsymbol{z_{ak}}$ of $\boldsymbol{z_k}$ can be found depending on the arbitrarily specified part $\boldsymbol{z_{bk}}$ and the task-specified part $\boldsymbol{\rho_{ak}}$.

However, *unlike* the PSFKs, the PSIKs need *not* be avoided, provided that the restricted specification $\boldsymbol{\rho_{ak}}$ is acceptable or even desirable for some special tasks.

This is because, in a pose of PSIK(*k*), $\boldsymbol{z_{bk}}$ can be realized by the actuator or actuators of the leg L_k and $\boldsymbol{\rho_{ak}}$ can be realized by the actuators of the other legs.

In other words, the manipulator remains under control in a pose of PSIK.

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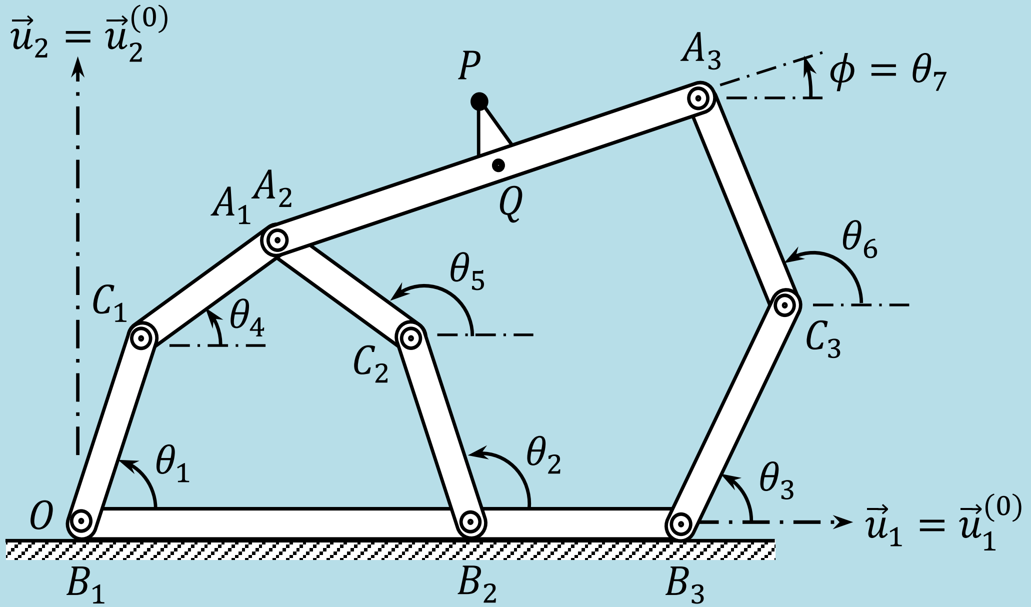
Example



A 3RRR Planar Parallel Manipulator

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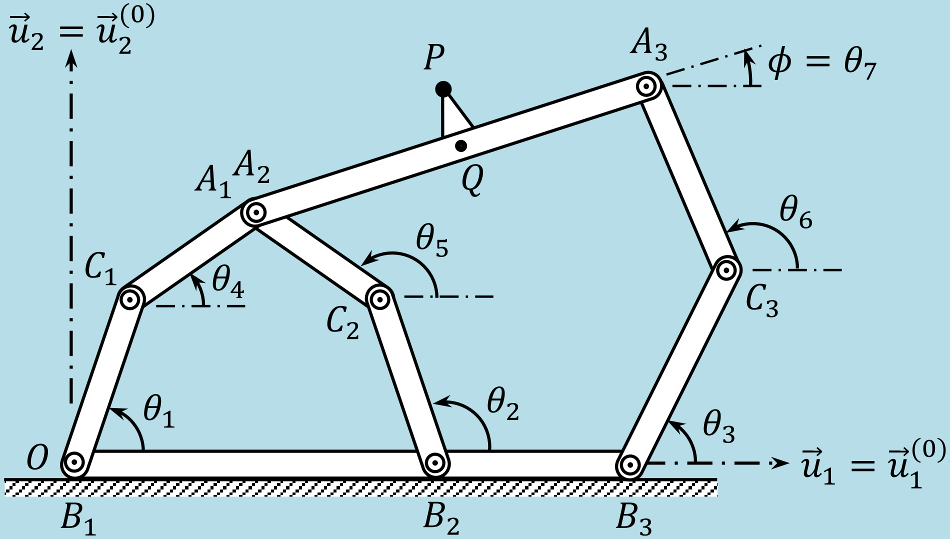
Independent Kinematic Loops



IKL-1 formed by L_1 and L_2 : $B_1B_2C_2A_2A_1C_1B_1$

IKL-2 formed by L_1 and L_3 : $B_1B_3C_3A_3A_2A_1C_1B_1$

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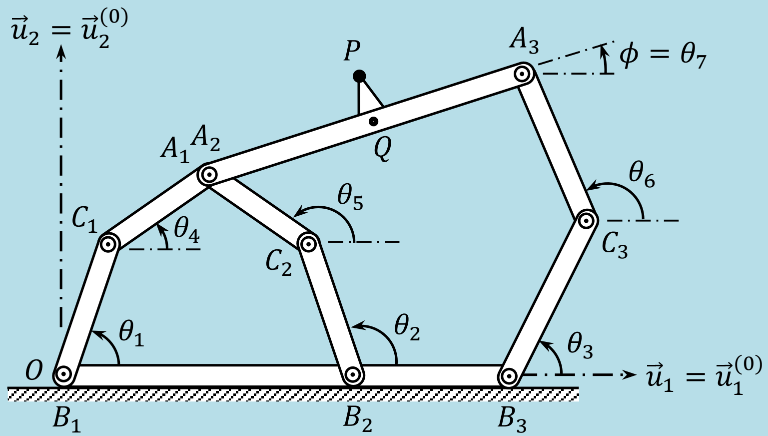
Kinematic Characteristics



Degree of Freedom

$$DoF = 3$$

Primary and Secondary Variables

$x_1 = \theta_1,\ x_2 = \theta_2,\ x_3 = \theta_3$ (Active Joint Variables)

$y_1 = \theta_4,\ y_2 = \theta_5,\ y_3 = \theta_6,\ y_4 = \theta_7 = \phi$ (Passive Joint Varaibles)

End-Effector Location Equations

$$\boldsymbol{p} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{y}) \Rightarrow \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} r_1 c\theta_1 + r_4 c\theta_4 + d_1 c\phi - h_7 s\phi \\ r_1 s\theta_1 + r_4 s\theta_4 + d_1 s\phi + h_7 c\phi \end{bmatrix}$$

Loop Closure Equations

$$\boldsymbol{\phi}(\boldsymbol{x}, \boldsymbol{y}) = \boldsymbol{0} \Rightarrow \begin{bmatrix} b_2 + r_2 c\theta_2 + r_5 c\theta_5 - r_1 c\theta_1 - r_4 c\theta_4 \\ r_2 s\theta_2 + r_5 s\theta_5 - r_1 s\theta_1 - r_4 s\theta_4 \\ b_3 + r_3 c\theta_3 + r_6 c\theta_6 - d_7 c\phi - r_1 c\theta_1 - r_4 c\theta_4 \\ r_3 s\theta_3 + r_6 s\theta_6 - d_7 s\phi - r_1 s\theta_1 - r_4 s\theta_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

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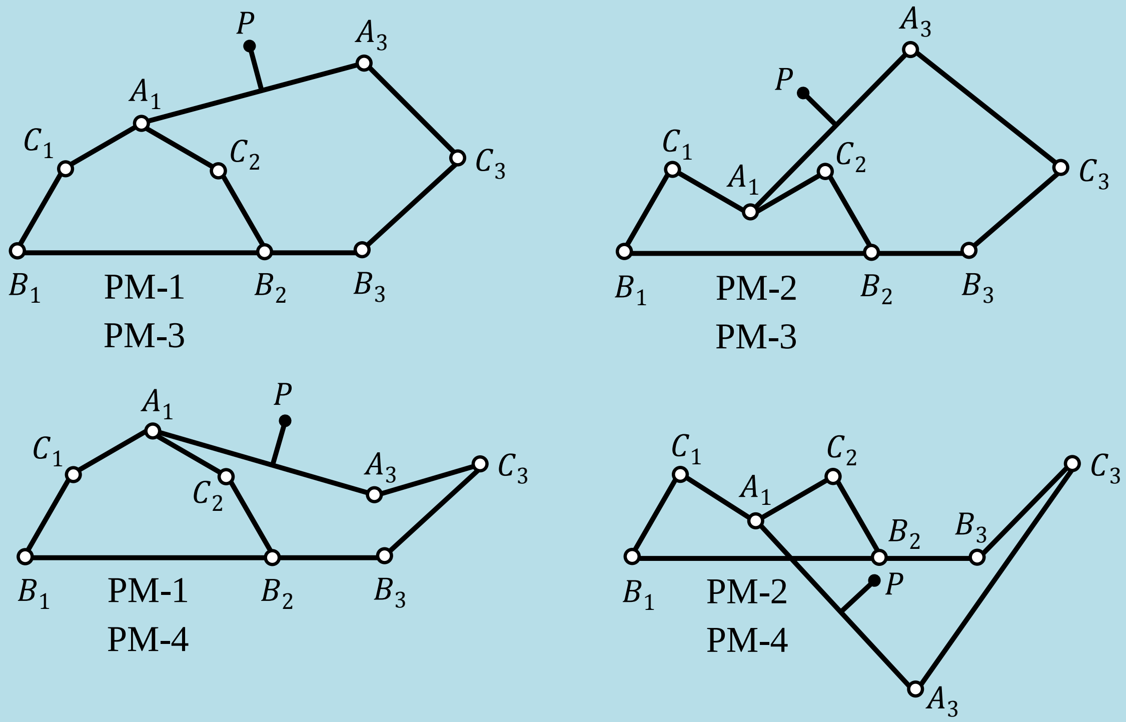
Posture Modes of Forward Kinematics (PMFKs)



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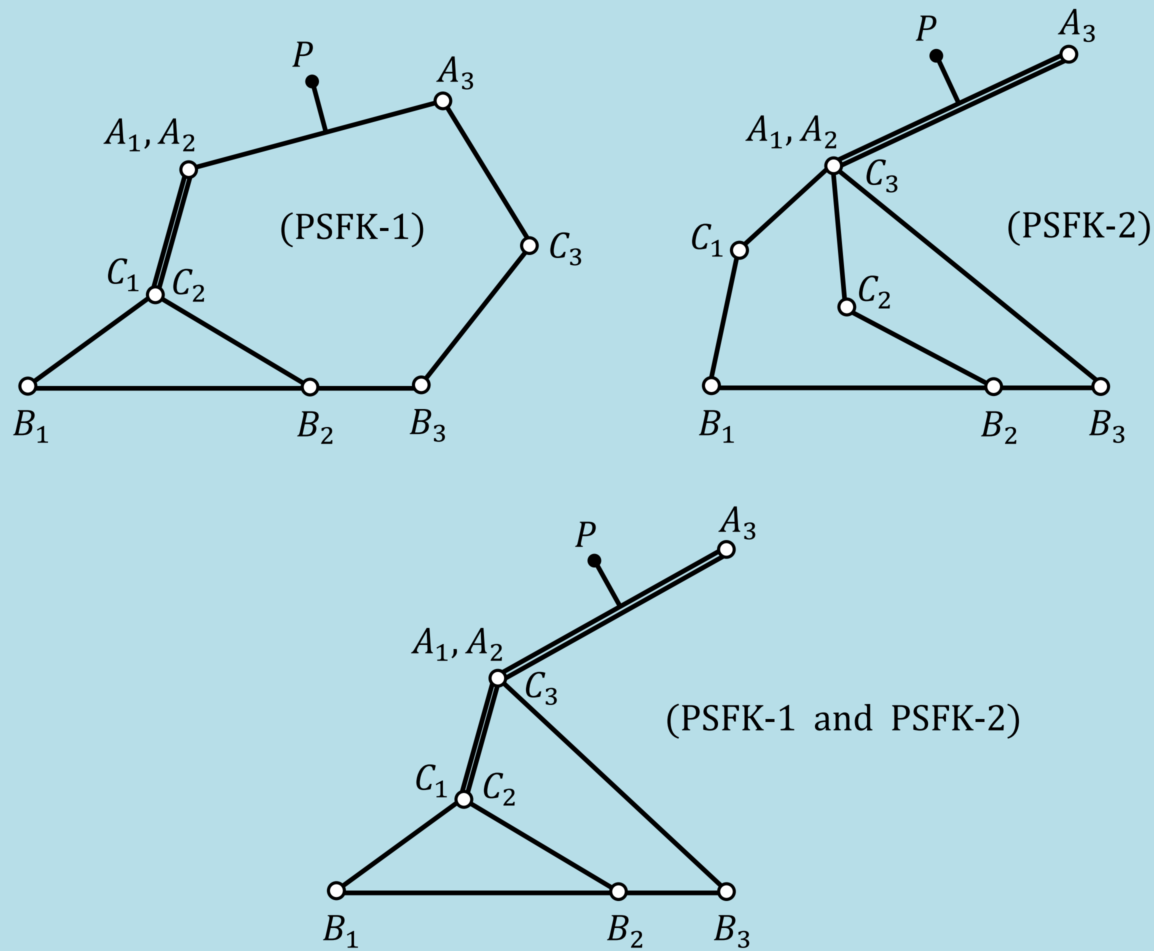
Position Singularities of Forward Kinematics (PSFKs)



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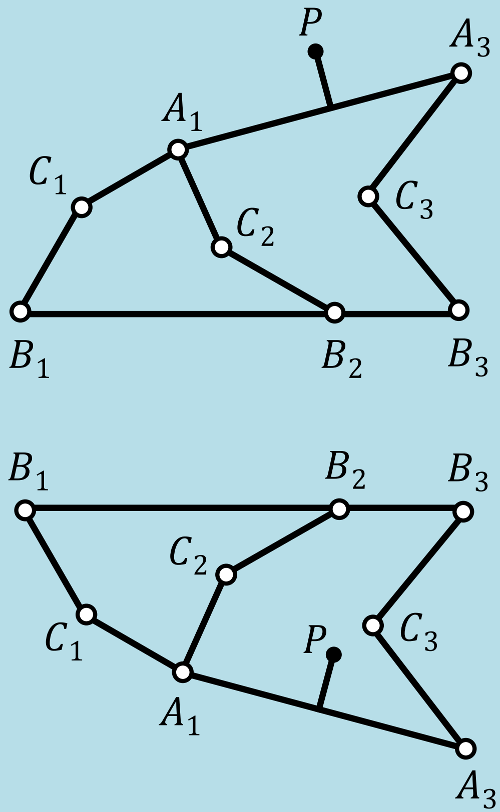
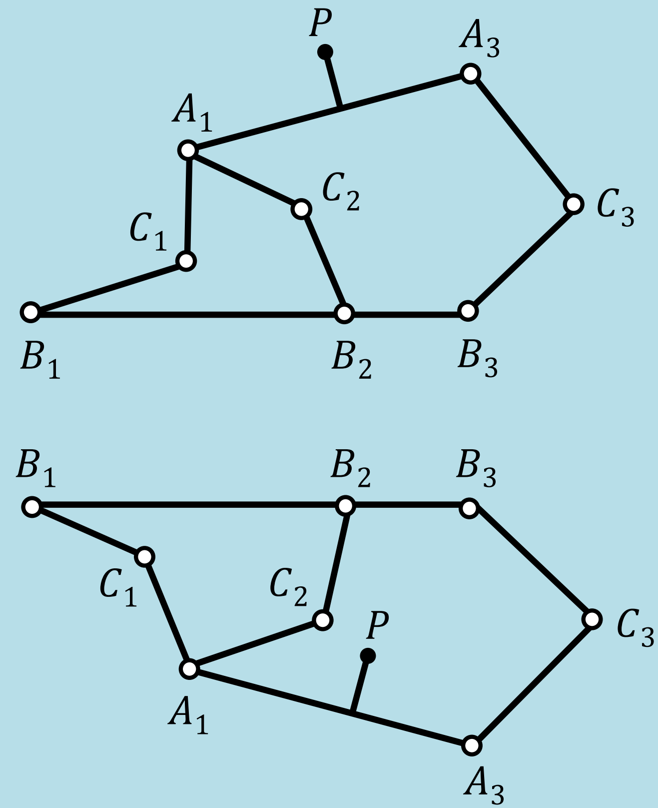
Posture Modes of Inverse Kinematics (PMIKs) For Two Different End-Effector Positions



Knee-Behind Posture Modes



Knee-Ahead Posture Modes

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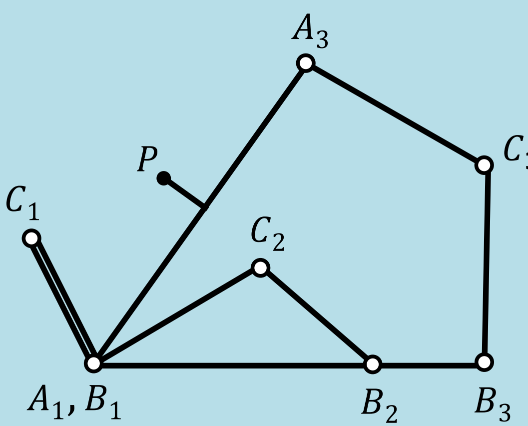
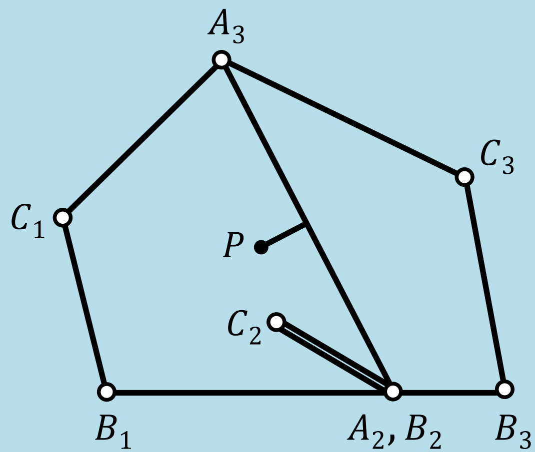
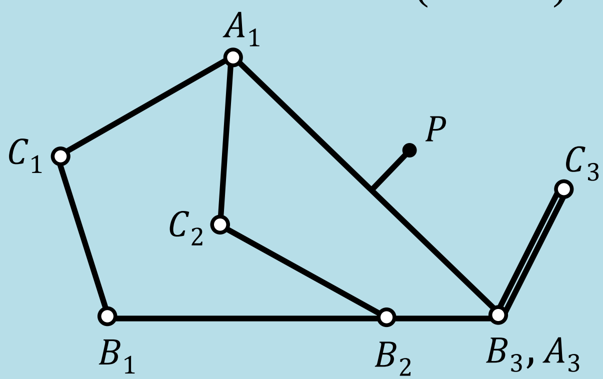
Position Singularities of Inverse Kinematics (PSIKs)



(PSIK-1)



(PSIK-2)



(PSIK-3)

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Forward Velocity Analysis

Velocity State of the End-Effector Through the Legs

$$\boldsymbol{\eta} = \boldsymbol{F_k}(\boldsymbol{x}, \boldsymbol{y})\dot{\boldsymbol{x}} + \boldsymbol{G_k}(\boldsymbol{x}, \boldsymbol{y})\dot{\boldsymbol{y}} \quad \text{for } \; k = 1, 2, \dots, l$$

$\boldsymbol{\eta} = \begin{bmatrix} \boldsymbol{v} \\ \boldsymbol{\omega} \end{bmatrix}$: Indicates the *Translational* and *Angular* Velocity of the End-Effector

$\boldsymbol{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$: Consists of the *Tip Point Velocity Components*

$\boldsymbol{\omega} = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}$: Consists of the *Angular Velocity Components* of the End-Effector

Forward Velocity Analysis

<u>Velocity Equations Written for the Independent Kinematic Loops</u>

$$\boldsymbol{\Psi}(\boldsymbol{x},\boldsymbol{y})\dot{\boldsymbol{y}} = \boldsymbol{\Phi}(\boldsymbol{x},\boldsymbol{y})\dot{\boldsymbol{x}}$$

$$\boldsymbol{\Psi}(\boldsymbol{x},\boldsymbol{y}) = \begin{bmatrix} \Psi_1(x,y) \\ \Psi_2(x,y) \\ \vdots \\ \Psi_{l-1}(x,y) \end{bmatrix} = \begin{bmatrix} G_2(x,y) - G_1(x,y) \\ G_3(x,y) - G_1(x,y) \\ \vdots \\ G_l(x,y) - G_1(x,y) \end{bmatrix}$$

$$\boldsymbol{\Phi}(\boldsymbol{x},\boldsymbol{y}) = \begin{bmatrix} \Phi_1(x,y) \\ \Phi_2(x,y) \\ \vdots \\ \Phi_{l-1}(x,y) \end{bmatrix} = \begin{bmatrix} F_1(x,y) - F_2(x,y) \\ F_1(x,y) - F_3(x,y) \\ \vdots \\ F_1(x,y) - F_l(x,y) \end{bmatrix}$$

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Forward Velocity Analysis

Forward Kinematic Solution

Given: The Rates of the Primary Variables (i.e. $\dot{\boldsymbol{x}}$)

Find: The Velocity State of the End-Effector (i.e. $\boldsymbol{\eta}$)

By-Product: The Rates of the Secondary Variables (i.e. $\dot{\boldsymbol{y}}$)

Stage 1

Find $\dot{\boldsymbol{y}}$ in terms of $\dot{\boldsymbol{x}}$ from the *velocity closure equations* of the IKLs.

$$\dot{\boldsymbol{y}} = [\boldsymbol{\Psi}(\boldsymbol{x},\boldsymbol{y})]^{-\mathbf{1}}\boldsymbol{\Phi}(\boldsymbol{x},\boldsymbol{y})\dot{\boldsymbol{x}} = \boldsymbol{H}(\boldsymbol{x},\boldsymbol{y})\dot{\boldsymbol{x}}$$

This solution will be valid, if $\boldsymbol{\Psi}(\boldsymbol{x},\boldsymbol{y})$ is not singular so that

$$\det[\boldsymbol{\Psi}(\boldsymbol{x},\boldsymbol{y})] \neq \mathbf{0}$$

Stage 2

Find $\boldsymbol{\eta}$ from one of the *leg-through velocity equations.*
If the leg L_k is selected for this purpose, then $\boldsymbol{\eta}$ will be:

$$\boldsymbol{\eta} = \boldsymbol{F}_{\boldsymbol{k}}(\boldsymbol{x},\boldsymbol{y})\dot{\boldsymbol{x}} + \boldsymbol{G}_{\boldsymbol{k}}(\boldsymbol{x},\boldsymbol{y})\boldsymbol{H}(\boldsymbol{x},\boldsymbol{y})\dot{\boldsymbol{x}} = \boldsymbol{J}(\boldsymbol{x},\boldsymbol{y})\dot{\boldsymbol{x}}$$

Note-1

The above formula for $\boldsymbol{\eta}$ is *indifferent to the selected leg*, because every leg-through velocity equation leads to the same velocity state of the end-effector.

Note-2

The above formula is also valid if $\det[\boldsymbol{\Psi}(\boldsymbol{x},\boldsymbol{y})] \neq \mathbf{0}$ due to the presence of $\boldsymbol{H}(\boldsymbol{x},\boldsymbol{y})$.

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Forward Velocity Analysis

<u>MSFKs (Motion Singularities of Forward Kinematics)</u>

In a pose of MSFK, a certain part of $\dot{\boldsymbol{x}}$ becomes constrained and therefore cannot be specified as desired. That is, the parts $\dot{\boldsymbol{x}}_{\boldsymbol{b}}$ and $\dot{\boldsymbol{x}}_{\boldsymbol{a}}$ of $\dot{\boldsymbol{x}}$ become tied up with a *singularity relationship* such as

$$\dot{\boldsymbol{x}}_{\boldsymbol{b}} = \boldsymbol{S}(\boldsymbol{x}, \boldsymbol{y})\dot{\boldsymbol{x}}_{\boldsymbol{a}}$$

Consequently, a corresponding part $\dot{\boldsymbol{y}}_{\boldsymbol{b}}$ of $\dot{\boldsymbol{y}}$ becomes indefinite, i.e. it cannot be found from the velocity closure equations.

This indefiniteness is transmitted to the position of the end-effector, too. That is, a part $\boldsymbol{\eta}_{\boldsymbol{b}}$ of $\boldsymbol{\eta}$ also becomes indefinite, i.e. uncontrollable by the actuators of the manipulator.

Therefore, the poses of MSFK must be avoided.

It happens that, *unlike* the PSFK poses, the MSFK poses are not effected by the geometric design parameters. So, they can be avoided only by controlling the actuators so that the singularity relationship is never allowed to occur.

Inverse Velocity Analysis

Inverse Velocity Solution

Given: The Velocity State of the End-Effector (i.e. $\boldsymbol{\eta}$)

Find: The Rates of Both Primary and Secondary Variables (i.e. $\dot{\boldsymbol{x}}$ and $\dot{\boldsymbol{y}}$)

Solution Method

The solution is obtained *leg by leg* from the *leg-through velocity equations*, which are written as

$$\boldsymbol{\eta} = \boldsymbol{F}_{\boldsymbol{k}}(\boldsymbol{x},\boldsymbol{y})\dot{\boldsymbol{x}}_{\boldsymbol{k}} + \boldsymbol{G}_{\boldsymbol{k}}(\boldsymbol{x},\boldsymbol{y})\dot{\boldsymbol{y}}_{\boldsymbol{k}} \text{ for } k = 1, 2, \dots, l$$

This equation can also be written as

$$\boldsymbol{\eta} = [\boldsymbol{F}_{\boldsymbol{k}}(\boldsymbol{x},\boldsymbol{y}) \quad \boldsymbol{G}_{\boldsymbol{k}}(\boldsymbol{x},\boldsymbol{y})] \begin{bmatrix} \dot{\boldsymbol{x}}_k \\ \dot{\boldsymbol{y}}_k \end{bmatrix} = \boldsymbol{T}_{\boldsymbol{k}}(\boldsymbol{x},\boldsymbol{y})\dot{\boldsymbol{z}}_{\boldsymbol{k}} \text{ for } k = 1, 2, \dots, l$$

Here, $\dot{\boldsymbol{x}}_{\boldsymbol{k}}$ and $\dot{\boldsymbol{y}}_{\boldsymbol{k}}$ are the parts of $\dot{\boldsymbol{x}}$ and $\dot{\boldsymbol{y}}$ that belong to the leg L_k.

If $\det[\boldsymbol{T}_{\boldsymbol{k}}(\boldsymbol{x},\boldsymbol{y})] \neq 0$, then $\dot{\boldsymbol{z}}_{\boldsymbol{k}}$ can be obtained as

$$\dot{\boldsymbol{z}}_{\boldsymbol{k}} = [\boldsymbol{T}_{\boldsymbol{k}}(\boldsymbol{x},\boldsymbol{y})]^{-\boldsymbol{1}}\boldsymbol{\eta}$$

Hence, $\dot{\boldsymbol{x}}_{\boldsymbol{k}}$ and $\dot{\boldsymbol{y}}_{\boldsymbol{k}}$ also become available.

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Inverse Velocity Analysis

<u>MSIKs (Motion Singularities of Inverse Kinematics)</u>

Since the inverse velocity solution is obtained leg by leg, a leg L_k may get into a pose MSIK, i.e. MSIK(*k*), *independently* of the other legs. Any two legs, e.g. L_i and L_j, may get into poses of MSIK(*i*) and MSIK(*j*) either simultaneously or at different times.

In a pose of MSIK(*k*), a certain part of $\boldsymbol{\eta}$ becomes constrained and therefore cannot be specified as desired. That is, the parts $\boldsymbol{\eta}_{bk}$ and $\boldsymbol{\eta}_{ak}$ of $\boldsymbol{\eta}$ become tied up with a *singularity relationship* such as

$$\boldsymbol{\eta}_{bk} = \boldsymbol{S}_k(\boldsymbol{x}, \boldsymbol{y})\boldsymbol{\eta}_{ak}$$

Consequently, a corresponding part $\dot{\boldsymbol{z}}_{bk}$ of $\dot{\boldsymbol{z}}_k$ becomes indefinite, i.e. it cannot be found from the leg-through velocity equation of the leg L_k and it can be specified arbitrarily as desired in addition to the freely specifiable part $\boldsymbol{\eta}_{ak}$ of $\boldsymbol{\eta}$.

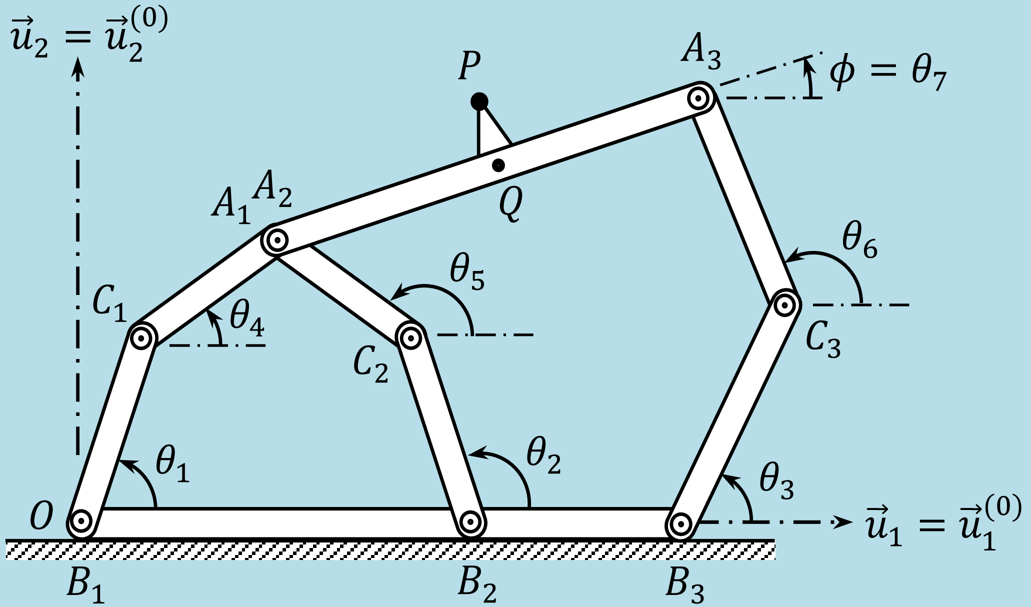
On the other hand, the complementary part $\dot{\boldsymbol{z}}_{ak}$ of $\dot{\boldsymbol{z}}_k$ can be found depending on the arbitrarily specified part $\dot{\boldsymbol{z}}_{bk}$ and the task-specified part $\boldsymbol{\eta}_{ak}$.

However, *unlike* the MSFKs, the MSIKs need *not* be avoided, provided that the restricted specification $\boldsymbol{\eta}_{ak}$ is acceptable or even desirable for some special tasks.

This is because, in a pose of MSIK(*k*), $\dot{\boldsymbol{z}}_{bk}$ can be realized by the actuator or actuators of the leg L_k and $\boldsymbol{\eta}_{ak}$ can be realized by the actuators of the other legs.

In other words, the manipulator remains under control in a pose of MSIK.

Example



A 3RRR Planar Parallel Manipulator

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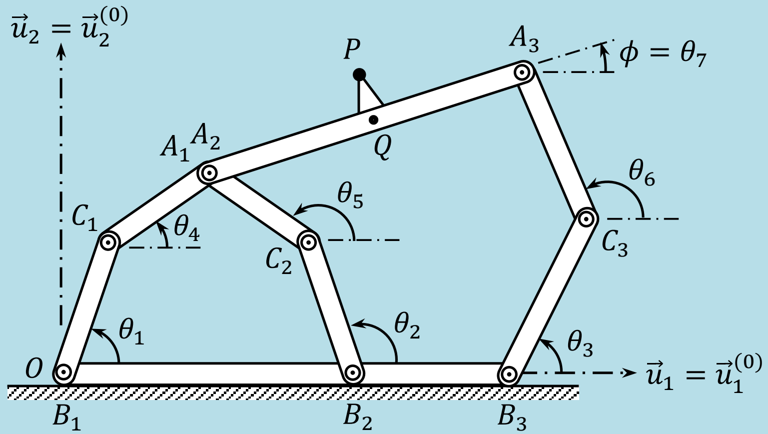
Kinematic Characteristics



<u>End-Effector Velocity Equations</u>

$$\boldsymbol{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} -r_1\dot{\theta}_1 s\theta_1 - r_4\dot{\theta}_4 s\theta_4 - d_1\dot{\phi} s\phi - h_7\dot{\phi} c\phi \\ r_1\dot{\theta}_1 c\theta_1 + r_4\dot{\theta}_4 c\theta_4 + d_1\dot{\phi} c\phi - h_7\dot{\phi} s\phi \end{bmatrix}$$

<u>Velocity Closure Equations</u>

$$\frac{d}{dt}\begin{bmatrix} b_2 + r_2 c\theta_2 + r_5 c\theta_5 - r_1 c\theta_1 - r_4 c\theta_4 \\ r_2 s\theta_2 + r_5 s\theta_5 - r_1 s\theta_1 - r_4 s\theta_4 \\ b_3 + r_3 c\theta_3 + r_6 c\theta_6 - d_7 c\phi - r_1 c\theta_1 - r_4 c\theta_4 \\ r_3 s\theta_3 + r_6 s\theta_6 - d_7 s\phi - r_1 s\theta_1 - r_4 s\theta_4 \end{bmatrix} = \mathbf{0}$$

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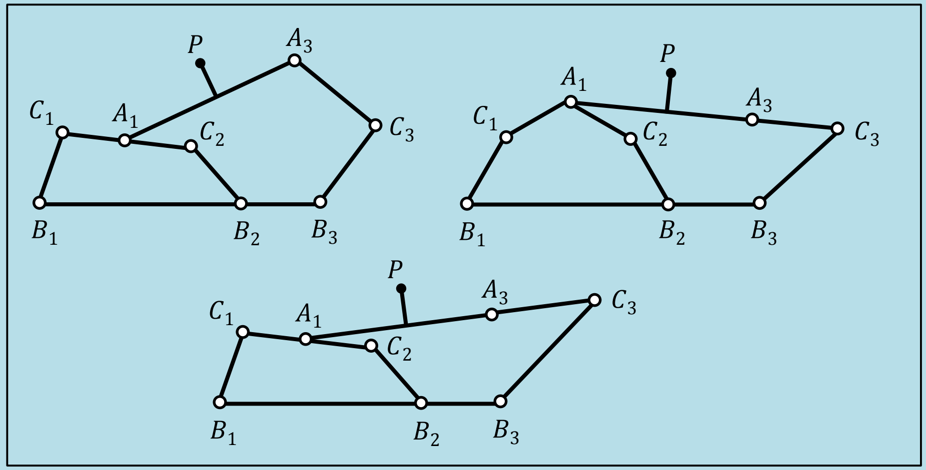
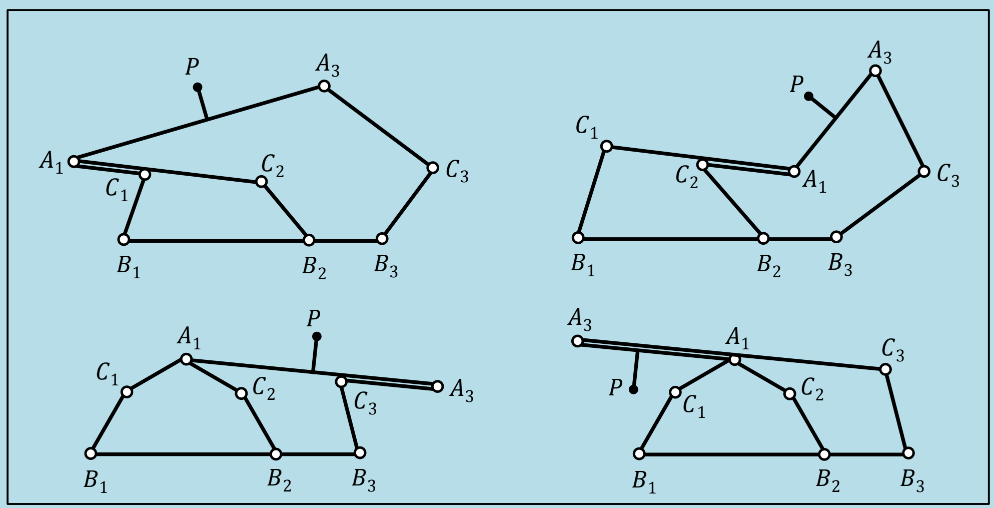
Motion Singularities of Forward Kinematics (MSFKs)

Extended MSFK Poses



Folded MSFK Poses



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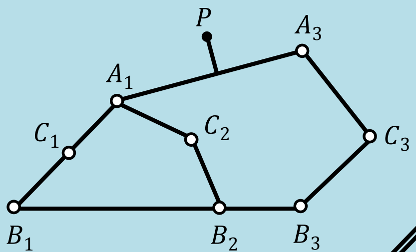
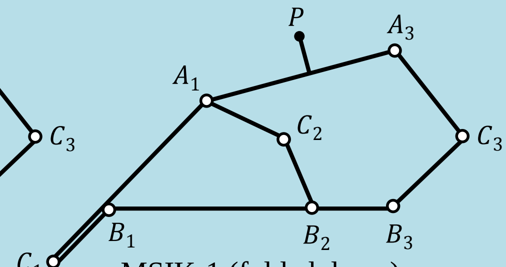
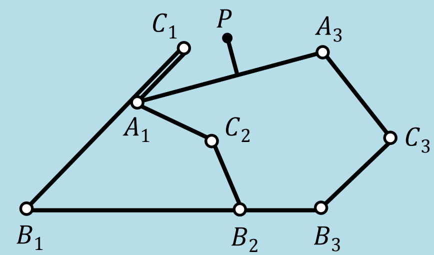
Motion Singularities of Inverse Kinematics (MSIKs)



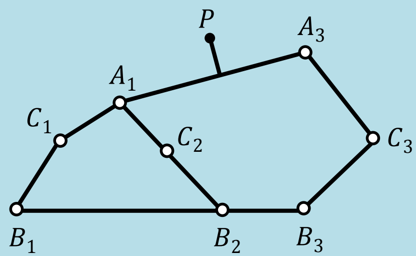
MSIK-1 (extended)



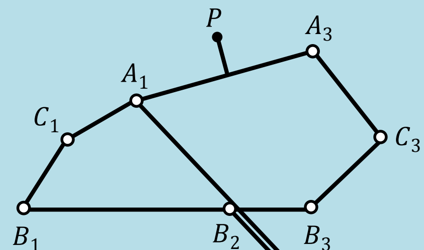
MSIK-1 (folded down)



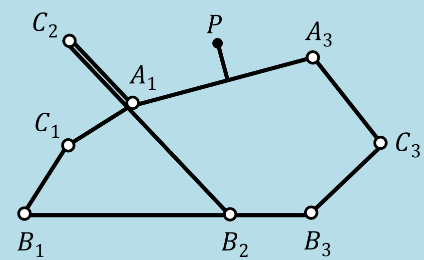
MSIK-1 (folded up)



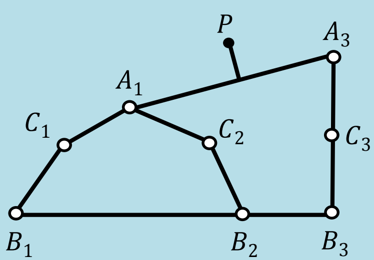
MSIK-2 (extended)



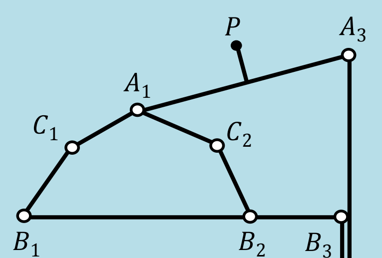
MSIK-2 (folded down)



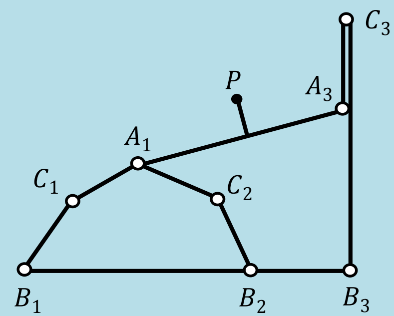
MSIK-2 (folded up)



MSIK-3 (extended)



MSIK-3 (folded down)



MSIK-3 (folded up)

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Thank you all
for
attending this presentation.

For more details, you may see the following publication:

Ozgoren, M. K., "Kinematic and Kinetostatic Analysis of Parallel Manipulators with Emphasis on Position, Motion, and ActuationSingularities", *Robotica*, Cambridge University Press, Vol. 37, No. 4, pp. 599-625, April 2019.

M. Kemal Özgören